DIGITAL SIGNAL PROCESSING
(ELEC 3141)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) The sampling frequency of the signal $x(t) = 4\sin(200\pi t) + 2\cos(50\pi t)$ should be
(a) Greater or equal to 100 Hz (b) Greater or equal to 200 Hz
(c) Greater or equal to 50 Hz (d) Greater or equal to 250 Hz.

(ii) If a signal $x(n)$ having N samples is convoluted with $h(n)$ having M samples then the after linear convolution the obtained signal will have
(a) (M+N-1) samples (b) Minimum (M,N) samples
(c) Maximum (M,N) samples (d) (M-N+1) samples.

(iii) The inverse z-transform of $\frac{3z}{z-2}, |z| > 2$ is
(a) $2(3)^n u(-n-1)$ (b) $3(2)^n u(-n-1)$
(c) $3(2)^n u(n)$ (d) $2(3)^n u(n)$.

(iv) If $\mathcal{F}\{x(n)\} = X(e^{j\omega})$ then $\mathcal{F}^{-1}\{e^{-j3\omega}X(e^{j\omega})\}$ will be
(a) $x(n)$ (b) $x(n-3)$ (c) $x(n+3)$ (d) $x(-n)$.

(v) The complex valued twiddle factor, W_N can be represented as,
(a) $e^{-j2\pi n}$ (b) $e^{-j2\pi/N}$ (c) $e^{-j2\pi}$ (d) $e^{-j2\pi nkN}$.

(vi) The condition for symmetry of impulse response of FIR system is
(a) $h(n) = h(N-1)$ (b) $h(n) = h(N+1)$
(c) $h(n) = h(N-n)$ (d) $h(n) = h(N-1-n)$.

(vii) In bilinear transformation the analog system with transfer function $H(s) = \frac{0.2}{s+0.9}$ is transformed to a digital system with transfer function,
(a) $H(s) = \frac{0.2}{\frac{2}{T}(\frac{1+z^{-1}}{1-z^{-1}})+0.9}$ (b) $H(s) = \frac{0.2}{\frac{T(1-z^{-1})}{2(1+z^{-1})}+0.9}$
(c) $H(s) = \frac{0.2}{\frac{2}{T}(\frac{1-z^{-1}}{1+z^{-1}})+0.9}$ (d) $H(s) = \frac{0.2}{\frac{T(1+z^{-1})}{2(1-z^{-1})}+0.9}$.

(viii) The inverse DFT of $X(k)$ can be expressed as

(a) $x(n) = \frac{1}{N}\sum_{k=0}^{N} X(k)e^{\frac{j2\pi kn}{N}}$ (b) $x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{\frac{j2\pi kn}{N}}$

(c) $x(n) = \frac{1}{N}\sum_{n=0}^{N-1} X(n)e^{-\frac{j2\pi kn}{N}}$ (d) $x(n) = N\sum_{n=0}^{N-1} X(k)\, e^{-\frac{j2pkn}{N}}$.

(ix) The ROC of the z-transform of the sequence $x(n) = u(-n)$is

(a) $|z| > 1$ (b) $|z| < 1$ (c) $-1 < |z| < 1$ (d) no ROC.

(x) The structure that uses separate delays for input and output samples is

(a) Direct form-I (b) Direct form-II (c) Cascade (d) Parallel.

Fill in the blanks with the correct word

(xi) To avoid the aliasing phenomenon, we can employ ________ filter

(xii) If $x(n) = \left\{1, \underset{\uparrow}{2}, 5, 4\right\}$ then $x(-n+3)$ is ________.

(xiii) The direct form-II realisation of N^{th} order system requires ________ delay.

(xiv) The Z-transform of $nu(n)$ is ________.

(xv) The transfer function of a 1^{st} order normalised Butterworth filter is ________.

Group - B

2. (a) Explain aliasing phenomenon. What do you mean by Nyquist rate?

[(CO1)(Remember/LOCQ)]

(b) Examine whether the following signal $g(n)$ is an energy or a power signal.

$g(n) = \left(\frac{1}{2}\right)^n u(n)$ *[(CO1)(Analyse/IOCQ)]*

(c) Determine $y(n) = x\left(\frac{n}{2}+3\right)$ and $z(n) = x\left(n-\frac{1}{2}\right)$, when $x(n) = \left\{3, \underset{\uparrow}{2}, 1, 4\right\}$.

[(CO1)(Analyse/IOCQ)]

(2 + 2) + 3 + (2 + 3) = 12

3. (a) Evaluate the output of the system whose impulse response is given by $h(n) = \left\{1, 4, \underset{\uparrow}{2}, 5\right\}$ for an input $x(n) = \left\{2, 4, \underset{\uparrow}{1}, 6\right\}$ by convolution technique.

[(CO1)(Evaluate /HOCQ)]

(b) Examine whether the following system is linear or not $y(n) = 3x(n) + \frac{1}{x(n-4)}$.

[(CO1)(Analyse/IOCQ)]

(c) Examine whether the following system is time invariant or not $y(n) = x(-n-4)$. *[(CO1)(Analyse/IOCQ)]*

(d) Solve the difference equation described by,

$C(k+3) + 5C(k+2) - 2C(k+1) + 3C(k) = u(k)$

Given that $C(0) = 2$, $C(1) = 1$, $C(2) = 0$ Consider $C(k)$ as output and $u(k)$ as input to the system which is a unit step signal. *[(CO1)(Analyse/IOCQ)]*

4 + 2 + 2 + 4 = 12

Group - C

4. (a) Determine the z-transform and ROC of the discrete time signal $x(n) = (0.875)^n\ u(-n-1) + (0.125)^n\ u(n)$. [(CO2)(Analyze/IOCQ)]

(b) Determine the inverse z-transform of $X(z) = \frac{1}{1-8z^{-1}+12z^{-2}}$, if ROC $|z| < 2$. [(CO2)(Analyze/IOCQ)]

(c) Evaluate the impulse response $h(n)$ for the system described by the 2nd order difference equation $y(n) - 0.6y(n-1) + 0.05y(n-2) = x(n)$, where $x(n)$ and $y(n)$ are the input and output sequence of the system. [(CO2)(Evaluate /HOCQ)]

4 + 4 + 4 = 12

5. (a) Obtain the expression of bilinear transformation to transform analog system to digital system. [(CO2)(Analyze/IOCQ)]

(b) Explain the mapping of s-plane to z-plane in bilinear transformation. What is frequency warping? [(CO2)(Understand/LOCQ)]

(c) For the analog transfer function $H(s) = \frac{16}{s^2+16}$, evaluate $H(z)$ using bilinear transformation if sampling time $T = 1$sec. [(CO2)(Evaluate/HOCQ)]

4 + (3 + 1) + 4 = 12

Group - D

6. (a) Determine the DTFT of the sequence $x(n) = u(n) - u(n-N)$. [(CO3)(Analyze/IOCQ)]

(b) Evaluate 4-point DFT of the sequence $x(n) = cos\frac{n\pi}{4}$ using matrix method. [(CO3)(Evaluate/HOCQ)]

(c) Evaluate inverse DFT of $X(k) = \{3, 2+j, 1, 2-j\}$. [(CO3)(Evaluate/HOCQ)]

4 + 4 + 4 = 12

7. (a) Given $x(n) = n + 1$, compute 8-point DFT of $x(n)$ using DIT-FFT algorithm. [(CO3)(Evaluate /HOCQ)]

(b) Compare the number of multiplications and additions required to compute the DFT of a 1024-point sequence using direct computation and that using FFT. [(CO3)(Understand/LOCQ)]

9 + 3 = 12

Group - E

8. Obtain the direct form-I, direct form-II and parallel form realization of the digital IIR filter represented by the transfer function $H(z) = \frac{3(2z^2+5z+4)}{(2z+1)(z+2)}$. [(CO4)(Analyze/IOCQ)]]

(4 + 4 + 4) = 12

9. (a) What is FIR system? Explain why FIR filter is always stable. [(CO4)(Remember /LOCQ)]

(b) Design a linear phase FIR low pass filter with a cut-off frequency, $\omega_c = 0.25\pi$rad/samples using 7 samples of Hamming window sequence. [(CO4)(Evaluate /HOCQ)]

(2 + 2) + 8 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	15.6	45.8	38.6

Course Outcome (CO):

After the completion of the course students will be able to

CO1: Understand, interpret, represent, manipulate, process, and analyze of discrete time signals and systems in the context of digital signal processing.

CO2: Understand a new representation of signal sequences with the z-transform, concept of transfer-function, and an application of z-transform properties for modeling of discrete time signals and stability analysis of systems.

CO3: Understand the frequency domain analysis of discrete time signals, spectral analysis and existence of efficient and fast algorithms for DSP systems.

CO4: Understand the design and analyze for digital filters, concept of linear-phase filters, realization of filter structures, mapping from analog filter to digital filter, and implementation of digital filters in real time (with Digital signal processor).

**LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.*