DISCRETE MATHEMATICS
(CSEN 2102)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) A connected planar graph having 5 vertices determines 3 regions. The number of edges of the graph is
(a) 6 (b) 5
(c) 2 (d) 4.

(ii) Let G and G^* be two graphs dual to each other. Which one of the following equals the number of edges of G?
(a) The number of edges of G^*
(b) The number of vertices of G^*
(c) The number of regions determined by G^*
(d) The number of components of G^*.

(iii) The number of pendant vertices of a binary tree with 48 edges is
(a) 25 (b) 24
(c) 23 (d) 49.

(iv) Let p be a prime number and a be any integer not divisible by p. Then $a^{p-1} + (p-1)! \equiv$
(a) $1\ (mod\ p)$ (b) $-1\ (mod\ p)$
(c) $2\ (mod\ p)$ (d) $0\ (mod\ p)$.

(v) "If x then y unless z" is represented by which of the following formula in propositional logic?
(a) $(x \wedge \sim z) \rightarrow y$ (b) $(x \wedge y) \rightarrow \sim z$
(c) $x \rightarrow (y \rightarrow \sim z)$ (d) $(x \rightarrow y) \wedge \sim z$.

(vi) The remainder in the division of $1! + 2! + 3! + 4! + 5! + 6! + \cdots + 100!$ by 3 is
(a) 1 (b) 2 (c) 0 (d) 4.

(vii) Let $a^2 \equiv b^2 (mod\ m)$, then which of the following is *not* true?
(a) $a^4 \equiv b^4 (mod\ m)$ (b) $a^{10} \equiv b^{10} (mod\ m)$
(c) $a^{20} \equiv b^{20} (mod\ m)$ (d) $a \equiv b (mod\ m)$.

(viii) The generating function of sequence $2, -2, 2, -2, 2, -2, 2, -2$ is
(a) $\frac{2}{1-z}$ (b) $\frac{2}{1+z}$ (c) $\frac{2}{(1-z)^2}$ (d) $\frac{2}{(1+z)^2}$.

(ix) There are 3 copies each of 4 different books. The number of ways of arranging them on a shelf is
(a) 12! (b) $\frac{12!}{3!}$ (c) $\frac{12!}{(3!)^4}$ (d) $\frac{12!}{3^4}$.

(x) The number of permutations of the letters of the word $PROGRESS$ is
(a) 40320 (b) 5040 (c) 10080 (d) 20160.

Fill in the blanks with the correct word

(xi) The number of edges of the complete bipartite graph $K_{3,4}$ is _____.

(xii) A connected graph is Eulerian if and only if it has no vertex of _____ degree.

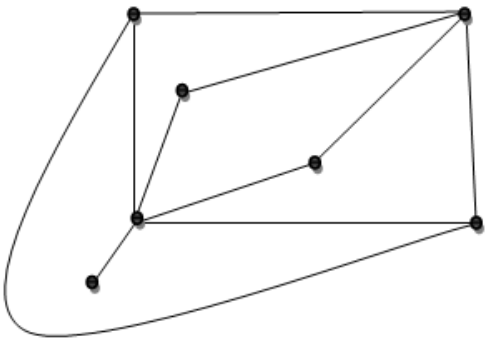
(xiii) The number of ways 6 people can sit around a round table is _____.

(xiv) Contrapositive of '$\sim p \rightarrow \sim q$' is _____.

(xv) The greatest common divisor of two consecutive Fibonacci numbers is_____.

Group - B

2. (a) Verify face-size equation for the following planar graph. Further verify Euler's formula.



[(CSEN2102.1,CSEN2102.2)(Apply/IOCQ)]

(b) Define maximal matching, maximum matching and perfect matching of a graph. Find all maximal matchings and the matching number for the graph C_7 (cycle graph with seven vertices). *[(CSEN2102.1,CSEN2102.2)(Remember/LOCQ)]*

(c) Write the chromatic polynomial of a tree with n vertices.
[(CSEN2102.1,CSEN2102.2)(Remember/LOCQ)]

(3 + 3) + 5 + 1 = 12

3. (a) Let G be a simple connected planar graph having n vertices, e edges and f regions. Prove that $2e \geq 3f$. *[(CSEN2102.1,CSEN2102.2)(Understand/LOCQ)]*

(b) How many ways can a tree having 5 vertices be coloured with at most 4 colours. Justify your answer. *[(CSEN2102.1,CSEN2102.2)(Apply/IOCQ)]*

(c) Prove that the non-planar graph with the smallest number of vertices is the complete graph having 5 vertices. *[(CSEN2102.1,CSEN2102.2)(Analyse/IOCQ)]*

6 + 3 + 3 = 12

Group - C

4. (a) Use the Euclidean algorithm to solve the Diophantine equation $34x + 21y = 2$.
[(CSEN2102.3)(Evaluate/HOCQ)]

(b) By mathematical induction, prove that $1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$.
[(CSEN2102.3)(Apply/IOCQ)]

6 + 6 = 12

5. (a) Use the Chinese Remainder theorem to solve the following system of congruences:

$$x \equiv 4(mod\ 5)$$
$$x \equiv 2\ (mod\ 6)$$
$$x \equiv (mod\ 7).$$

[(CSEN2102.3)(Apply/IOCQ)]

(b) Let a be an odd integer. Prove that 24 is a divisor of $a(a^2 - 1)$.
[(CSEN2102.3)(Analyse/IOCQ)]

6 + 6 = 12

Group - D

6. (a) Use the method of generating function to solve the following recurrence relation:
$a_n = 3a_{n-1} + 1; n \geq 1$, given that $a_0 = 1$. *[(CSEN2102.4)(Create/HOCQ)]*

(b) There are 21 consonants and 5 vowels in the English alphabet. Consider only 8-letter words with 3 different vowels and 5 different consonants.
(i) How many such words can be formed?
(ii) How many such words contain the letter d?
(iii) How many contain the letter a and b? *[(CSEN2102.4)(Analyse/IOCQ)]*

6 + (2 × 3) = 12

7. (a) Prove Newton's Identity: $C(n,r)C(r,k) = C(n,k)C(n-k,r-k)$ for integers $n \geq r \geq k \geq 0$. *[(CSEN2102.4)(Analyse/IOCQ)]*

(b) Find the number of nonnegative integral solutions of the equation $x_1 + x_2 + x_3 + \cdots + x_{10} = 8$ by showing in detail that this number is the same as the number of binary numbers with nine 1's and eight 0's. *[(CSEN2102.4)(Evaluate/HOCQ)]*

6 + 6 = 12

Group - E

8. (a) Construct the truth table for the following compound proposition:
$\big(p \to (q \to r)\big) \to ((p \to q) \to (p \to r))$. *[(CSEN2102.5,CSEN2102.6)(Understand/LOCQ)]*

(b) Prove that the premises $a \to (b \to c), d \to (b \wedge \sim c)$ and $a \wedge d$ are inconsistent.
[(CSEN2102.5, CSEN2102.6)(Analyse/IOCQ)]

(c) Write the dual of $(p \vee q) \to r$. *[(CSEN2102.5,CSEN2102.6)(Remember/IOCQ)]*

5 + 5 + 2 = 12

9. (a) Find the CNF of $\sim(p \vee q) \leftrightarrow (p \wedge q)$ *[(CSEN2102.5,CSEN2102.6)(Evaluate/HOCQ)]*

(b) Prove that the following argument is valid without using a truth table.
$p \longrightarrow (q \longrightarrow s),\ \sim r \vee p,\ q \implies r \longrightarrow s.$ *[(CSEN2102.5,CSEN2102.6)(Analyse/IOCQ)]*

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	17.71	57.29	25

Course Outcome (CO):

After the completion of the course students will be able to

CSEN2102. 1. Interpret the problems that can be formulated in terms of graphs and trees.

CSEN2102. 2. Explain network phenomena by using the concepts of connectivity, independent sets, cliques, matching, graph coloring etc.

CSEN2102. 3. Achieve the ability to think and reason abstract mathematical definitions and ideas relating to integers through concepts of well-ordering principle, division algorithm, greatest common divisors and congruence.

CSEN2102. 4. Apply counting techniques and the crucial concept of recurrence to comprehend the combinatorial aspects of algorithms.

CSEN2102. 5. Analyze the logical fundamentals of basic computational concepts.

CSEN2102. 6. Compare the notions of converse, contrapositive, inverse etc in order to consolidate the comprehension of the logical subtleties involved in computational mathematics.

**LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.*