METHODS IN OPTIMIZATION
(MATH 4121)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) Which of the following Hessian matrices is negative definite?
(a) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 2 & 2 \\ -1 & 4 \end{pmatrix}$ (d) $\begin{pmatrix} -2 & 3 \\ 0 & 1 \end{pmatrix}$

(ii) Let $Q(x, y, z)$ be a quadratic form such that $Q(1,2,3) = 5$ and $Q(-1, -2, -3) = -5$, then
(a) $Q(x, y, z)$ could be indefinite
(b) $Q(x, y, z)$ could be positive definite
(c) $Q(x, y, z)$ could be negative semi definite
(d) $Q(x, y, z)$ could be positive semi definite.

(iii) The optimal solution of the following game problem is

PLAYER B

PLAYER A		B_1	B_2	B_3	B_4
	A_1	-5	3	1	20
	A_2	5	5	4	6
	A_3	-4	-2	0	-5

(a) (A_3, B_2) (b) (A_2, B_2) (c) (A_1, B_1) (d) (A_2, B_3)

(iv) The bordered Hessian matrix of the Lagrangian function L constructed for an optimization problem with equality constraints is given by

$$H^B(L) = \begin{pmatrix} 0 & 2 & 4 & 1 \\ 2 & 0 & 3 & 2 \\ 4 & 3 & 3 & 1 \\ 1 & 2 & 1 & 3 \end{pmatrix}$$

then the number of independent variables, the objective function depending on is,
(a) 1 (b) 2 (c) 3 (d) 0

(v) Out of the following search algorithms, which one has the fastest rate of convergence?
(a) Interval halving method (b) Dichotomous search
(c) Fibonacci search (d) Golden section search.

(vi) For the standard maximization problem, the Kuhn-Tucker necessary conditions are also sufficient conditions, provided
(a) objective function is convex and constraints are concave
(b) objective function and constraints are concave
(c) objective function and constraints are convex
(d) objective function is concave and constraints are convex.

(vii) The distinguishing feature of an LP model is
(a) relationship among all variables is linear
(b) it has only one constraint
(c) decision variables are negative
(d) the objective function is always maximization type.

(viii) If two constraints in an LPP do not intersect in the positive quadrant of the graph then
(a) the problem is infeasible (b) the solution is unbounded
(c) one of the constraints is redundant (d) the solution is unique.

(ix) The method used for solving an assignment problem is called
(a) reduced matrix method (b) MODI method
(c) Hungarian method (d) Simplex method.

(x) For a maximization problem, the objective function coefficient for an artificial variable is (where $M > 0$)
(a) $+M$ (b) $-M$ (c) 0 (d) $1/M$

Fill in the blanks with the correct word

(xi) If a function is concave on a __________ domain then any local maximum is global maximum.

(xii) In a ________ game gains of one player are equal to the losses of other player.

(xiii) The value of the golden ratio is ___________.

(xiv) In the Golden section search algorithm, if the initial interval of uncertainty is L_0, then the length of the final interval L_j is ______________________.

(xv) A feasible solution to a transportation problem is said to be ________ if and only if the corresponding cells in the transportation table do not contain a loop.

Group - B

2. (a) A firm plans to purchase at least 200 quintals of scrap containing high quality metal x and low quality metal y. It decides that the scrap to be purchased must contain at least 100 quintal of x-metal and not more than 35 quintals of y-metal. The firm can purchase the scrap from two suppliers (A and B) in unlimited quantities. The percentage of x and y metals in terms of weight in the scraps supplied by A and B is given below.

Metals	Supplier A	Supplier B
x	25%	75%
y	10%	20%

Formulate this problem as an L.P.P. model and hence solve it by graphical method to find the minimum quantity of scrap to be purchased from suppliers A and B. *[(MATH4121.1, MATH4121.2)(Create/HOCQ)]*

(b) Solve the following L.P.P. by simplex method:

Maximize $Z = 7x_1 + 5x_2$

subject to the constraints

$$x_1 + 2x_2 \leq 6$$
$$4x_1 + 3x_2 \leq 12$$
$$x_1, x_2 \geq 0.$$

[(MATH4121.1,MATH4121.2)(Apply/IOCQ)]

6 + 6 = 12

3. (a) Solve the following L.P.P. by Big-M method:

Maximize $Z = 12x_1 + 20x_2$

subject to the constraints

$$6x_1 + 8x_2 \geq 100$$
$$7x_1 + 12x_2 \leq 120$$
$$x_1,\ x_2 \geq 0.$$

[(MATH4121.1,MATH4121.2)(Apply/IOCQ)]

(b) Give the dual of the L.P.P.:

Minimize $Z = 2x_1 + 3x_2 + 4x_3$

subject to the constraints

$$2x_1 + 3x_2 + 3x_3 \geq 2$$
$$3x_1 + x_2 + 7x_3 = 3$$
$$x_1 + 4x_2 + 6x_3 \leq 5$$
$$x_1,\ x_2 \geq 0 \text{ and } x_3 \text{ is unrestricted.}$$

[(MATH4121.1,MATH4121.2)(Understand/LOCQ)]

7 + 5 = 12

Group - C

4. (a) Use graphical method in solving the following game and find the value of the game.

PLAYER B

PLAYER A	2	4
	2	3
	3	2
	−2	6

[(MATH4121.1, MATH4121.2, MATH4121.3, MATH4121.4)(Apply/IOCQ)]

(b) Solve the following game, by dominance rule:

PLAYER B

PLAYER A	−1	−1	2	1
	2	2	0	1
	3	−2	1	−2
	2	1	-3	2

[(MATH4121.1,MATH4121.2,MATH4121.3,MATH4121.4)(Understand/LOCQ)]

6 + 6 = 12

5. (a) Solve the given transportation problem to find the optimal solution.

	I	II	III	*IV*	Supply
A	15	10	17	18	2
B	16	13	12	13	6
C	12	17	20	11	7
Demand	3	3	4	5	

[(MATH4121.1,MATH4121.2,MATH4121.3,MATH4121.4)(Evaluate/HOCQ)]

(b) Solve the assignment problem where the cost of assigning operators **I to IV** to machines **A to D** is given below:

	A	**B**	**C**	**D**
I	1	4	6	3
II	9	7	10	9
III	4	5	11	7
IV	8	7	8	5

[(MATH4121.1,MATH4121.2MATH4121.3,MATH4121.4)(Evaluate/HOCQ)]

7 + 5 = 12

Group - D

6. (a) Use Kuhn-Tucker conditions to solve the following non-linear programming problem:
Maximize $Z = 10x_1 - x_1^2 + 10x_2 - x_2^2$
subject to the constraints

$$x_1 + x_2 \leq 9$$
$$x_1 - x_2 \geq 6$$
$$x_1, x_2 \geq 0.$$

[(MATH4121.6)(Evaluate/HOCQ)]

(b) Determine the relative maximum and minimum (if any) of the following function:

$$f(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2 - 4x_1 - 8x_2 - 12x_3 + 56.$$

[(MATH4121.6)(Understand/LOCQ)]

8 + 4 = 12

7. Solve the following non-linear programming problem using the method of Lagrangian multipliers method:
Optimize $Z = x_1^2 - 10x_1 + x_2^2 - 6x_2 + x_3^2 - 4x_3$
subject to the constraints

$$x_1 + x_2 + x_3 = 7$$
$$x_1, x_2, x_3 \geq 0.$$

[(MATH4121.6)(Evaluate/HOCQ)]

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Group - E

8. Use interval halving method to minimize $f(x) = x^5 - 5x^3 - 20x + 5$ over $[0,\ 5]$ taking the tolerance to be less than 0.4.

[(MATH4121.1,MATH4121.5)(Apply/IOCQ)]

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9. Use Golden Section search algorithm to minimize $f(x) = x^2(x - 2.5)$ in $[0,\ 1]$ taking the stopping tolerance to be less than 0.2.

[(MATH4121.1,MATH4121.5)(Apply/IOCQ)]

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Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	15.62	44.79	39.59

Course Outcome (CO):

After the completion of the course students will be able to
MATH4121.1 Describe the way of writing mathematical model for real-world optimization problems.
MATH4121.2 Identify Linear Programming Problems and their solution techniques.
MATH4121.3 Categorize Transportation and Assignment problems.
MATH4121.4 Apply the way in which Game Theoretic Models can be useful to a variety of real-world scenarios in economics and in other areas.
MATH4121.5 Apply various optimization methods for solving realistic engineering problems and compare their accuracy and efficiency.
MATH4121.6 Convert practical situations into non-linear programming problems and solve unconstrained and constrained programming problems using analytical techniques

**LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.*