ENGINEERING COMPUTATIONAL TECHNIQUES
(MECH 4124)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) Numerical techniques more commonly involve
(a) Elimination method (b) Reduction method
(c) Iterative method (d) Direct method.

(ii) A mathematical model of a physical system or process is
(a) formulation of equations (b) description of the system
(c) experimentation with the system (d) explanation of the result.

(iii) The number of significant digits in 3.0509 is
(a) 3 (b) 5
(c) 6 (d) 7.

(iv) Which of the following is also known as the Newton-Raphson method?
(a) Chord method (b) Tangent method
(c) Diameter method (d) Secant method.

(v) The definite integral $\int_a^b f(x)dx$ can be interpreted as
(a) Area below the curve $y = f(x)$ from a to b
(b) Area above the curve $y = f(x)$ from a to b
(c) Difference in end point values, $f(b) - f(a)$
(d) Arithmetic average, $\frac{1}{2}\{f(b) + f(a)\}$.

(vi) Match the items in columns I and II:

Column I	Column II
A. Gauss-Seidel method	1. Numerical integration
B. Newton-Raphson method	2. Ordinary differential equations
C. Runge-Kutta method	3. Root finding procedure
D. Simpson's Rule	4. Solution of system of linear algebraic equations

(a) A-1, B-4, C-3, D-2 (b) A-1, B-4, C-2, D-3
(c) A-1, B-3, C-2, D-4 (d) A-4, B-3, C-2, D-1.

(vii) In Trapezoidal rule, the result of integration is equal to sum of the functional values multiplied with

(a) h/2 (b) h/3 (c) 3h/8 (d) h/4.

(viii) The motion of a particle $x(t)$ moving along the x −axis, whose velocity at time t is specified by the continuous function $f(t)$ and whose initial position is specified as $x(t = t_0) = x_0$, is described as

(a) $x(t) = \int_{t_0}^{t} f(\tau)d\tau$ (b) $x(t) = x_0 + \int_{t_0}^{t} f(\tau)d\tau$

(c) $x(t) = \int_{t}^{t_0} f(\tau)d\tau$ (d) $x_0 = x(t) + \int_{t_0}^{t} f(\tau)d\tau$.

(ix) A differential equation is classified as 'ordinary' if it has

(a) one dependent variable (b) more than one dependent variable

(c) one independent variable (d) more than one independent variable.

(x) Which of the following equations is parabolic?

(a) $f_{xy} - f_x = 0$ (b) $f_{xx} + 2f_{xy} + f_{yy} = 0$

(c) $f_{xx} + 2f_{xy} + 4f_{yy} = 0$ (d) None of the above equations.

Fill in the blanks with the correct word

(xi) The deflection $w(x)$ of the free end of a flexible beam in response to a load force $q(x)$ is described by the ODE $\frac{d^4w}{dx^4} + a^2\frac{d^2w}{dx^2} = q(x)$. The order of this ODE is _________.

(xii) The values of a function $f(x)$ are tabulated below:

x:	0	1	2	3
f(x):	1	2	1	10

Using Newton's forward difference formula, the cubic polynomial that can be fitted to the above data is _________.

(xiii) Starting from $x_0 = 1$, one step of Newton-Raphson method in solving the equation $x^3 + 3x - 7 = 0$ gives the next value (x_1) as _________.

(xiv) Consider the first-order system $x' = x - 2y;\; y' = -2x + y$. The second-order ODE equivalent to this system is _________.

(xv) The equation $(\partial^2u/\partial x^2 + \partial^2u/\partial y^2) = f(x, y)$ is known as _________.

Group - B

2. (a) Find a root of the equation $x^3 - 4x - 9 = 0$ using the Bisection method correct to three decimal places. *[(CO2)(Apply/IOCQ)]*

(b) Find a real root of the equation $x^2 - 2x - 5 = 0$ by the Regula-falsi method correct to two decimal places. *[(CO2)(Apply/IOCQ)]*

6 + 6 = 12

3. (a) Find the real root of the equation $3x = \cos x + 1$ correct to two decimal places, using the Newton-Raphson method. *[(CO2)(Apply/IOCQ)]*

(b) Find the real root of the equation $x^3 + x - 1 = 0$ correct to two decimal places, using the Newton-Raphson method. *[(CO2)(Analyze/IOCQ)]*

6 + 6 = 12

Group - C

4. (a) Apply the Gauss elimination method to solve the equations $x + 4y - z = -5;\ x + y - 6z = -12;\ 3x - y - z = 4.$ *[(CO2)(Apply/IOCQ)]*

(b) Fit a polynomial of degree three which takes the following values:

x:	3	4	5	6
y:	6	24	60	120

[(CO3)(Analyze/IOCQ)]

6 + 6 = 12

5. (a) Apply the Gauss-Seidel iteration method to solve the equations $2x - 3y + z = -1;\ x + 4y + 5z = 25;\ 3x - 4y + z = 2.$ upto third iteration. *[(CO2)(Apply/IOCQ)]*

(b) Use Lagrange's interpolation formula to fit a polynomial to the following data:

x:	1	2	3	4
y:	-8	3	5	7

Hence find y (0.5), y (2.5) and y (3.5). *[(CO3)(Analyze/IOCQ)]*

6 + 6 = 12

Group - D

6. (a) Evaluate the definite integral $\int_0^1 \frac{dx}{(1+x)}$ by using (i) Trapezoidal rule and (ii) Simpson's 1/3 rule. Divide the interval (0, 1) into four parts each of width h = 0.25. *[(CO4)(Apply/IOCQ)]*

(b) Using Euler's method, find an approximate value of y corresponding to x=1, given that $dy/dx = y - x$ and $y\ (x = 0) = 1$. *[(CO4) (Analyze/IOCQ)]*

6 + 6 = 12

7. (a) Using modified Euler's method, find an approximate value of y when x = 0.3, given that $dy/dx = x + y$ and $y\ (x = 0) = 1$. *[(CO5)(Apply/IOCQ)]*

(b) Using the Picard's method of successive approximation, obtain a solution upto the third approximation of the equation $dy/dx = y + x$ such that $y\ (x = 0) = 1$. *[(CO5) (Apply/IOCQ)]*

6 + 6 = 12

Group - E

8. (a) Using the Taylor series method, obtain the values of y at x=0.1 and at x=0.2 by solving the equation $dy/dx = x^2y + 1$ such that $y(x = 0) = 1$. *[(CO5)(Apply/IOCQ)]*

(b) Apply the Runge-Kutta fourth-order method to find an approximate value of y when x = 0.2, given that $dy/dx = x - y$ and $y\ (x = 0) = 1$. *[(CO5)(Analyze/IOCQ)]*

6 + 6 = 12

9. (a) Solve the elliptic equation $u_{xx} + u_{yy} = 0$ for the square mesh of Fig. 1 with boundary values as shown. Consider uniform step-size. Carry out computations till the 5th iteration level.

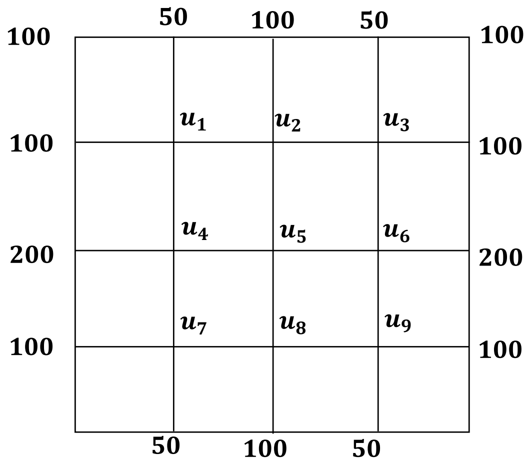

Fig. 1: Square mesh

[(CO6) (Investigate/HOCQ)]

(b) The linear partial differential equation $x^2 u_{xx} + (1 - y^2) u_{yy} = 0$ $(-\infty < x < \infty$, $-1<y<1$ is parabolic: Justify. *[(CO6)(Apply/IOCQ)]*

10 + 2 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	0	89.58	10.42

Course Outcome (CO):

After the completion of the course students will be able to

CO 1: Apply mathematical models for numerical solutions and classify different types of error.

CO 2: Solve a system of linear algebraic equations by different methods and find out the roots.

CO 3: Implement the regression and interpolation methods for curve fitting and solve different types of optimization problems.

CO 4: Use different numerical integration methods for practical problems.

CO 5: Classify Initial and Boundary value problems to select appropriate solution strategies, and solve Eigenvalue problems applied to physical systems.

CO 6: Apply the Finite Difference Method and the Finite Element Method to formulate and develop solutions for one-dimensional and two-dimensional problems in partial differential equations.

****LOCQ:*** *Lower Order Cognitive Question;* ***IOCQ:*** *Intermediate Order Cognitive Question;* ***HOCQ:*** *Higher Order Cognitive Question.*