DISCRETE MATHEMATICS
(MATH 2103)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) A graph has 15 vertices and 20 edges. The number of edges which should be removed from it in order to obtain a tree is
(a) 13 (b) 5
(c) 19 (d) 6.

(ii) A planar graph has 10 vertices, 6 edges and 3 regions. The number of components of the graph is
(a) 3 (b) 5
(c) 6 (d) 7.

(iii) The generating function of the sequence $2, 2, 2, 2, \ldots$ is
(a) $\frac{1}{1-x}$ (b) $\frac{2}{1-x}$
(c) $\frac{1}{(1-2x)^2}$ (d) $\frac{1}{1-2x}$

(iv) The number of permutations of the letters of the words BAT and $BALL$ are, respectively,
(a) 6 and 6 (b) 6 and 24
(c) 6 and 12 (d) 24 and 12.

(v) The greatest common divisor of the Fibonacci numbers F_{20} and F_{21} is
(a) 3 (b) 2
(c) 1 (d) 0

(vi) Let m be a positive integer. Then $(m^2+1)^3 \equiv$
(a) $2 \pmod{m}$ (b) $3 \pmod{m}$
(c) $0 \pmod{m}$ (d) $1 \pmod{m}$

(vii) The remainder in the division of $1! + 2! + 3! + \cdots + 200!$ by 6 is

(a) 1 (b) 2

(c) 3 (d) 4.

(viii) In the proposition, $p \vee (\sim p \vee q)$ is

(a) a tautology (b) a contradiction

(c) p (d) q

(ix) The square of an odd number is of the form

(a) $3k + 2$ (b) $3k + 1$

(c) $8k + 2$ (d) $8k + 1$

(x) A loop in G gives one of the following in its dual

(a) pendant edge (b) loop

(c) a pair of parallel edges (d) an even vertex.

Fill in the blanks with the correct word

(xi) The number of ways in which 6 people can sit around a round table is _________.

(xii) The chromatic number of K_8, the complete graph having 8 vertices is _________.

(xiii) In a group of 13 children, there must be at least ______________ children who were born in the same month.

(xiv) $\sim(p \vee q)$ is equivalent to _________.

(xv) A 5 -vertex colourable graph has three blocks. Then each block is ______________ vertex colourable.

Group - B

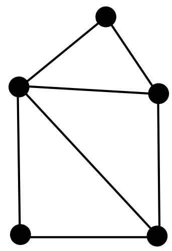
2. (a) Let G be a simple connected planar graph with n vertices, e edges and f regions. Then show that $e \leq 3n - 6$. *[(MATH2103.1, MATH2103.2)(Remember/LOCQ)]*

(b) Prove that if G is a simple planar graph then G has a vertex v such that $\deg(v) \leq 5$. *[(MATH 2103.1, MATH 2103.2)(Apply/IOCQ)]*

(c) Prove that Kuratowski's first graph is non-planar. *[(MATH 2103.1, MATH 2103.2)(Analyse/IOCQ)]*

4 + 4 + 4 = 12

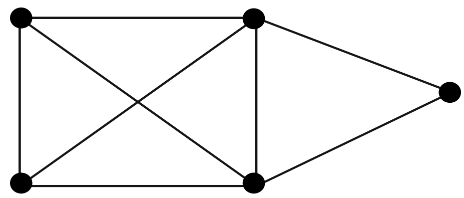
3. (a) Find the chromatic polynomial of the following graph by Decomposition Theorem:



[(MATH2103.1, MATH2103.2)(Evaluate/HOCQ)]

(b) Show that if a graph has at least one edge then the sum of the coefficients in its chromatic polynomial is zero. *[(MATH2103.1, MATH2103.2)(Apply/IOCQ)]*

(c) Find the minimum and maximum value of the chromatic number $\chi(G)$ of the following graph:



[(MATH2103.1, MATH2103.2)(Apply/IOCQ)]

6 + 3 + 3 = 12

Group - C

4. (a) Prove that the product of any m consecutive integers is divisible by m.

[(MATH 2103.),(Analyse/IOCQ)]

(b) Let a denote an integer. Prove that $3a^2 - 1$ is never a perfect square.

[(MATH 2103.3)(Analyse/IOCQ)]

6 + 6 = 12

5. (a) Let $a \equiv b \pmod{n}$. Prove that $a^4 \equiv b^4 \pmod{n}$. Does $a^4 \equiv b^4 \pmod{n}$ imply $a \equiv b \pmod{n}$? Justify your answer. *[(MATH2103.3)(Understand/LOCQ)]*

(b) Use the Chinese Remainder Theorem to solve the following system of congruences:

$$x \equiv 1 \pmod{3}$$
$$x \equiv 2 \pmod{5}$$
$$x \equiv 3 \pmod{7}.$$

[(MATH2103.3)(Analyze/IOCQ)]

6 + 6 = 12

Group - D

6. (a) The question paper of mathematics contains 10 questions divided into two groups of five questions each. In how many ways can an examinee answer 6 questions taking at least two questions from each group.

[(MATH 2103.4)(Understand/LOCQ)]

(b) Use the method of generating function to solve the recurrence relation
$a_n = 8a_{n-1} + 10^{n-1}, n \geq 2, a_0 = 1, a_1 = 9.$ *[(MATH2103.4)(Evaluate/HOCQ)]*

6 + 6 = 12

7. (a) A total of 1232 students have taken a course in Spanish, 874 have taken a course in French and 114 have taken a course in Russian. Further, 103 have taken courses in both Spanish and French, 23 have taken courses in both Spanish and Russian and 14 have taken courses in both French and Russian. If 2092 students have taken at least one of Spanish, French and Russian, how many students have taken a course in all three languages? *[(MATH2103.4)(Understand/LOCQ)]*

(b) In how many ways can the letters of the English alphabet be arranged so that there are exactly 5 letters between the letters *a* and *b*? Show your work in detail.

[(MATH2103.4)(Understand/LOCQ)]

7 + 5 = 12

Group - E

8. (a) Construct the truth table for the following compound proposition: $(p \leftrightarrow q) \vee (\sim q \leftrightarrow r)$. *[(MATH2103.5, MATH2103.6)(Remember/LOCQ)]*

   (b) Write down the converse, inverse and contrapositive of the following statement: If it rains, there is cloud in the sky. *[(MATH2103.5, MATH2103.6)(Analyze/IOCQ)]*

6 + 6 = 12

9. (a) Prove the following equivalence without using a truth table.
$((p \vee \sim q) \wedge (\sim p \vee \sim q)) \vee q \equiv T$ (Tautology) *[(MATH 2103.5, MATH 2103.6) (Evaluate/HOCQ)]*

   (b) Prove the validity of the following argument without using a truth table: "If I study, then I will pass in the examination. If I do not go to the picnic, then I will study. But I failed in the examination. Therefore, I went to the picnic." *[(MATH 2103.5, MATH 2103.6)(Evaluate/HOCQ)]*

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	35.42	39.58	25

Course Outcome (CO):

After the completion of the course students will be able to

MATH 2103.1. Interpret the problems that can be formulated in terms of graphs and trees.

MATH 2103.2. Explain network phenomena by using the concepts of connectivity, independent sets, cliques, matching, graph coloring etc.

MATH 2103.3. Achieve the ability to think and reason abstract mathematical definitions and ideas relating to integers through concepts of well-ordering principle, division algorithm, greatest common divisors and congruence.

MATH 2103.4. Apply counting techniques and the crucial concept of recurrence to comprehend the combinatorial aspects of algorithms.

MATH 2103.5. Analyze the logical fundamentals of basic computational concepts.

MATH 2103.6. Compare the notions of converse, contrapositive, inverse etc in order to consolidate the comprehension of the logical subtleties involved in computational mathematics.

**LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.*