PHYSICS - I
(PHY 1001)

Time Allotted: 2½ hrs **Full Marks: 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) An eigen function of an operator $\frac{d^2}{dx^2}$ is

(a) $cos\ ^2kx$ (b) e^{kx} (c) x^2 (d) lnx

(ii) If the states of a quantum system with Hamiltonian H at $t = 0$ and $t = t$ are given by $\varphi(0)$ and $\varphi(t)$ then

(a) $\varphi(t) = e^{-\frac{iHt}{\hbar}}\varphi(0)$ (b) $\varphi(t) = H\varphi(0)$
(c) $\varphi(t) = \varphi(0)$ (d) $\varphi(t) =$(-iHt/ħ)$\varphi(0)$

(iii) If right circularly polarized light is incident on a half wave plate therefore the output will be

(a) left circularly polarized (b) left elliptically polarized
(c) plane polarized (d) unpolarized.

(iv) The spacing between nth and (n+1) th energy levels i.e., ΔE_n for a free particle in a one-dimensional box is

(a) $\Delta E_n = nE_1$ (b) $\Delta E_n = (2n+1)E_1$
(c) $\Delta E_n = (n+1)E_1$ (d) $\Delta E_n = (n-1)E_1$, E_1 is the ground state energy.

(v) The path difference introduced due to the reflection of light wave from denser to rarer medium is

(a) λ (b) 2λ (c) $\frac{\lambda}{2}$ (d) $\frac{\lambda}{3}$

(vi) In Newton's ring experiment, if the air film in between plano-convex lens and the glass plate is replaced by a liquid of refractive index of μ, then the diameter of rings

(a) reduces in the ratio $\sqrt{\mu}$: 1 (b) increases in the ratio μ:1
(c) reduces in the ratio 1: $\sqrt{\mu}$ (d) increases in the ratio $\sqrt{\mu}$: 1

(vii) A damped oscillator is acted upon by an external force $F = F_0 cos\omega t$. If ω_0 is the natural frequency of oscillation, the frequency of oscillation of the system at steady state is

(a) ω_0 (b) ω (c) $\omega - \omega_0$ (d) $\omega + \omega_0$

(viii) Which of the following does not represent a central force?

(a) $\vec{F} = \frac{3}{r^2}\hat{\theta}$ (b) $\vec{F} = \frac{-5}{r^2}\hat{r}$ (c) $\vec{F} = \frac{1}{r^3}\hat{r}$ (d) $\vec{F} = 3r^2\hat{r}$

(ix) Two forced oscillating systems have identical quality factor. If their time-periods are 0.02s and 0.05s respectively, the ratio of their damping factors is
(a) $\frac{2}{5}$ (b) $\frac{5}{2}$ (c) $\frac{4}{5}$ (d) $\frac{2}{25}$

(x) In the five-point acceleration formula the Euler term is given by
(a) $\vec{\omega} \times (\vec{\omega} \times \vec{r})$ (b) $\frac{d\vec{\omega}}{dt}\Big]_{rot} \times \vec{r}$
(c) $-\frac{d\vec{\omega}}{dt}\Big]_{rot} \times \vec{r}$ (d) $-\vec{\omega} \times (\vec{\omega} \times \vec{r})$, The symbols have their usual meaning.

Fill in the blanks with the correct word

(xi) $|\varphi|^2$ gives the _____ of finding a particle in a state φ.

(xii) In single slit diffraction, if the slit width is increased, the width of the central maximum _____.

(xiii) The relation between relaxation time (τ) and logarithmic decrement (Δ) of a damped harmonic oscillator is _____.

(xiv) In quantum mechanics the value of $[\hat{x}, \widehat{p_x}] =$ _____.

(xv) For a series LCR circuit with AC voltage in resonance the current is _____ phase with the voltage.

Group - B

2. (a) Using Newton's equation of motion and definition of central force illustrate that the motion of a planet is always confined in a single plane. *[(CO5)(Understand/LOCQ)]*

(b) For a plane polar coordinate system justify the following relations: $\dot{\hat{r}} = \dot{\theta}\hat{\theta}$ and $\dot{\hat{\theta}} = -\dot{\theta}\hat{r}$. *[(CO5)(Evaluate/HOCQ)]*

(c) Starting from the differential equation of orbit justify that in absence of any external force a planet will move in a straight line orbit. *[(CO5)(Evaluate/HOCQ)]*

(d) From the fundamental definition of areal vector defend Kepler's second law. *[(CO5)(Evaluate/HOCQ)]*

3 + 3 + 3 + 3 = 12

3. (a) The position vector $\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath} + z(t)\hat{k}$ of a particle moving relative to the Earth surface is given by the following three equations: $\ddot{x} = 2\omega\dot{y}\cos\lambda$, $\ddot{y} = -2\omega\dot{z}\sin\lambda - 2\omega\dot{x}\cos\lambda$ and $\ddot{z} = -g + 2\omega\dot{y}\sin\lambda$. Here, ω is the Earth's angular velocity with respect to the Z axis of the fixed frame, λ is the co-latitude of the initial position of the particle and g is acceleration due to gravity when earth is not rotating. If the particle is thrown with an initial velocity v from rest along the longitude towards north, Show that it will be deflected towards east in the northern hemisphere. *[(CO5)(Understand/LOCQ)]*

(b) There are two reference frames S and S' whose origins are O and O' respectively. At $t = 0$, O and O' coincide. If O' starts moving with a constant acceleration $\vec{f}$ with respect to O then elaborate that Newton's equation of motion will not be identical for S and S'. *[(CO5)(Creating/HOCQ)]*

(c) A reference frame S' is rotating with respect to a fixed frame S with an angular velocity $\vec{\omega} = \omega\hat{k}$ while their Z axis coinciding. If the position vector of a particle of mass m with respect to S' is $\vec{r} = \alpha^2 t\hat{\imath}' + t^2\hat{\jmath}'$, Find the Coriolis force acting on the particle. Here, α is a positive constant. *[(CO5)(Remember/LOCQ)]*

5 + 3 + 4 = 12

Group - C

4. (a) Establish the differential equation of damped motion using energy conservation, where damping force is directly proportional to velocity of the particle. *[(CO1)(Remember/LOCQ)]*

   (b) In one dimensional motion of a mass of 1 kg. is acted upon by a restoring force 100 dyne / cm and a damping force of 10 dyne sec/cm.
   (i) Justify whether the motion is oscillatory or not?
   (ii) Find the value of relaxation time of the system. *[(CO1)(Remember/LOCQ)]*

   (c) A particle is undergoing a damped vibration subject to the condition of initial displacement x_0 and velocity is v_0. Find the solution of the vibration of the particle in critical damping condition. *[(CO1)(Analyse/IOCQ)]*

   (d) The frequency of an under damped harmonic oscillator is adjusted to be equal to half the frequency experienced by the oscillator without damping. Predict the logarithmic decrement of the system. *[(CO1)(Create/HOCQ)]*

3 + (1 + 1) + 4 + 3 = 12

5. (a) Write down the expression of the amplitude of a forced harmonic oscillator under external periodic force in its steady state explaining all terms. Draw the amplitude resonance plot with angular frequency of the external force for two different damping constants. Show that at velocity resonance, the maximum velocity is inversely proportional to damping factor. *[(CO1)(Remember/LOCQ)]*

   (b) The dynamics of a system undergoing forced oscillation is given by the equation of motion:

$$4\frac{d^2x}{dt^2} + 6\pi\frac{dx}{dt} + 10\pi^2 x = 5cos\alpha\pi t, \quad \alpha > 0$$

   Estimate the values of α, for which amplitude and velocity resonance is possible. Find the Q-factor of the system. *[(CO1)(Evaluate/LOCQ)]*

(2 + 2 + 3) + (3 + 2) = 12

Group - D

6. (a) Decide whether the following function represents a classical wave:
   $f(x,t) = A\sin(ax)\cos(bt)$, here A, a and b are positive constants.
   [(CO2)(Evaluate/HOCQ)]

   (b) When a thin film of transparent material is placed over one of the slits in Young's double slit experiment, the central bright fringe is displaced by 5 fringes. The refractive index of the material of the film is 1.4 and the wavelength of the light is 5500 Å.
   (i) What is the thickness of the film?
   (ii) What would probably be observed if the thickness of the film is 1 mm?
   [(CO3)(Remember/LOCQ)]

   (c) Construct an expression for the radius of m^{th} dark ring in a Newton's rings experiment. *[(CO2)(Apply/IOCQ)]*

4 + (2 + 2) + 4 = 12

7. (a) The light vector of a polarized light is given by $\vec{E}(z,t) = 3\cos(\omega t - kz)\hat{\imath} + 2\,cos\left(\omega t - kz + \frac{3\pi}{4}\right)\hat{\jmath}$. Determine the state of polarization of the light and draw the same. *[(CO3)(Evaluate/HOCQ)]*

(b) A screen is placed 2m away from a narrow slit. Find the slit width, if the first minimum lies 5 mm on either side of the central maximum when plane waves ($\lambda = 500\ nm$) are incident on the slit. *[(CO2)(Remember/IOCQ)]*

(c) A linearly polarized wave passes through a half-wave plate made of positive crystal with $n_0 = 1.5$. The minimum thickness of the plate is twice the wave length of light used. Solve the value of n_e. *[(CO3)(Apply/IOCQ)]*

(d) Starting from the expression of intensity of a single slit diffraction examine that the intensity of the 2nd secondary maxima formed by a single slit for diffraction process is nearly 1.65% of the principal maximum. *[(CO2)(Create/HOCQ)]*

4 + 3 + 2 + 3 = 12

Group - E

8. (a) Examine that the function $\varphi(x) = Ae^{-ikx} + Be^{ikx}$ is an eigenfunction of the kinetic energy operator $[\widehat{E_k} = \frac{\widehat{p_x^2}}{2m}]$ and also find its eigen value. *[(CO4)(Analyse/IOCQ)]*

(b) Decide that the momentum operator $[\widehat{P_x} = -i\hbar\frac{d}{dx}]$ is a Hermitian operator relative to the inner-product $\int_{a_1}^{a_2} \emptyset^*(x)\varphi(x)dx$ along with $\emptyset(a_1) = \emptyset(a_2) = \varphi(a_1) = \varphi(a_2) = 0$. *[(CO4)(Evaluate/HOCQ)]*

(c) Show that two quantum operators $z^2\frac{d^2}{dz^2}$ and $z\frac{d}{dz}$ have simultaneous eigenfunction. *[(CO4)(Understand/LOCQ)]*

(d) A quantum system has two normalized eigenstates φ_1 and φ_2. If the system is in a state $\varphi = \sqrt{\frac{2}{3}}\ \varphi_1 + \sqrt{\frac{1}{3}}\ \varphi_2$, estimate the probability of finding the system in the respective eigenstates. *[(CO6)(Create/HOCQ)]*

(2 + 1) + 3 + 3 + 3 = 12

9. (a) Staring from the time-dependent Schördinger equation, Construct the time-independent equation for a particle moving in one dimension under the influence of a potential V(x). *[(CO6)(Create/HOCQ)]*

(b) Show that $\varphi(x) = x(x-1)$ is an acceptable quantum state for a particle confined in a one dimensional box given by the equation where $0 \leq x \leq 1$.
(i) Determine the normalized states. (ii) Find the expectation value of energy.
(iii) Estimate the uncertainty in the measurement of position. *[(CO6)(Remember/LOCQ)]*

3 + (3 + 3 + 3) = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	46.87	16.66	36.47

Course Outcome (CO):

After the completion of the course students will be able to
1. Understanding physical systems in terms of their modelling of time evolution.
2. Comprehending wave interpretation of natural phenomena and Implications of allied observations
3. Understanding theoretical backgrounds associated to some experiments based on wave phenomena.
4. Grasping an analytic view of micro and macroscopic world.
5. Accessing the knowledge of the behaviour of a particle under the influence of different potential.
6. Understanding conservative systems based on their particle and wave nature.

**LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.*