MATHEMATICAL & STATISTICAL METHODS
(MATH 2101)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) If X follows has exponential distribution with parameter $\lambda = 1$, then $P(X > 3)$ is
(a) $\frac{1}{e^6}$ (b) e^3 (c) e . (d) $\frac{1}{e^3}$.

(ii) In Simpson's $\frac{1}{3}rd$ rule for finding the value of $\int_a^b f(x)dx$ there exists no error if $f(x)$ is a
(a) parabolic function (b) non-linear function
(c) logarithmic function (d) exponential function.

(iii) Fourier series for the function $f(x)$ in $(-l, l)$ is
(a) $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l}$; $f(x)$ is even in $(-l, l)$
(b) $f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$; $f(x)$ is odd in $(-l, l)$
(c) $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$
(d) All of these.

(iv) If X is a normal variate with mean -2 and variance 25 then which one of the following is standard normal variate
(a) $\frac{x+2}{25}$ (b) $\frac{x+2}{5}$ (c) $\frac{x-2}{25}$ (d) $\frac{x+5}{2}$.

(v) Newton's backward interpolation formula is used to interpolate a function $y = f(x)$ where interpolating point should be
(a) near end of the table (b) near centre of the table
(c) near beginning of the table (d) at any point in the table.

(vi) Order and degree of the partial differential equation $\frac{\partial^2 z}{\partial x^2} = 2 + \left(\frac{\partial z}{\partial x}\right)^{\frac{1}{3}}$ are
(a) 2, 2 (b) 3, 2 (c) 3, 3 (d) 2, 3.

(vii) Particular integral of $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial x^2} = \sin(2x + y)$ is

(a) $-\frac{\sin(2x+y)}{9}$ (b) $\frac{\sin(2x+y)}{9}$

(c) $-\frac{\sin(2x+y)}{15}$ (d) $-\sin(2x + y)$.

(viii) Lagrange's auxiliary equations of $p\tan^2 x + q\tan^2 y - \tan x \tan y = 0$ is

(a) $\frac{dx}{\tan^2 x} = \frac{dy}{\tan^2 y} = \frac{dz}{-\tan x \tan y}$ (b) $\frac{dx}{\tan^2 y} = \frac{dy}{\tan^2 x} = \frac{dz}{-\tan x \tan y}$.

(c) $\frac{dx}{\tan x} = \frac{dy}{\tan y} = \frac{dz}{1}$ (d) $\frac{dx}{\tan^2 x} = \frac{dy}{\tan^2 y} = \frac{dz}{\tan x \tan y}$.

where, $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$.

(ix) Considering $h = 1$, the value of $\Delta^2(e^x) =$

(a) $(e-1)e^x$ (b) $(e-1)^2 e^{2x}$

(c) $(e-1)^2 e^x$ (d) e^x.

(x) When two variables x and y are uncorrelated, then the correlation coefficient between them is

(a) 0 (b) ± 1 (c) 1 (d) -1 .

Fill in the blanks with the correct word

(xi) The degree of the interpolating polynomial for a function $f(x)$ whose values are known at 8 interpolating points is ________________ .

(xii) In trapezoidal rule, to evaluate $\int_2^9 f(x)dx$ taking seven equal sub-intervals, the value of the step length (h) is __________________ .

(xiii) The mode of the following observation 5, 7, 5, 2, 6, 2, 7, 4, 2, 6, 3 is ______________ .

(xiv) $z = (x + px)^2$ is a first order _________ partial differential equation.

(xv) In the Fourier series expansion of $f(x) = x$ in $[-\pi, \pi]$, the value of a_0 is ________.

Group - B

2. (a) Apply Lagrange's method to solve $\frac{y^2 z}{x} p + xzq = y^2$ where, $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$.

[(MATH2101.5)(Apply/IOCQ)]

(b) Solve $\left(D^2 - 3DD' + 2D'^2\right)z = e^{2x-y} + cos(x + 2y)$ where $D \equiv \frac{\partial}{\partial x}, D' \equiv \frac{\partial}{\partial y}$.

[(MATH2101.5)(Understand/LOCQ)]

6 + 6 = 12

3. (a) Find a complete integral of $px + qy = pq$ using Charpit's method.

[(MATH2101.5) (Apply/IOCQ)]

(b) Obtain a partial differential equation by eliminating ϕ from the relation $z = \phi\left(\frac{y}{x}\right)$.

[(MATH2101.5)(Create/HOCQ)]

8 + 4 = 12

Group - C

4. (a) Use Simpson's $\frac{1}{3}$rd rule to evaluate $\int_0^1 \frac{xdx}{(1+x)}$ taking ten equal sub-intervals, correct up to three decimal places. *[(MATH2101.1,MATH2101.2)(Remember/LOCQ)]*

(b) Using Newton's forward interpolation formula find the value of y when $x = 12$ using the following table:

x	10	15	20	25	30	35
y	35.3	32.4	29.2	26.1	23.2	20.5

[(MATH2101.1,MATH2101.2)(Analyze/IOCQ)]

6 + 6 = 12

5. (a) Use Lagrange's interpolation formula to find the value of $f(x)$ at $x = 0$, given

x	-1	-2	2	4
$f(x)$	-1	-9	11	69

[(MATH2101.1,MATH2101.2)(Understand/LOCQ)]

(b) Find the value of $\int_0^1 e^x dx$ by Trapezoidal rule with $h = 0.1$. Hence, find the value of the absolute error in the solution. *[(MATH2101.1,MATH2101.2)(Apply/IOCQ)]*

6 + 6 = 12

Group - D

6. (a) Find the Fourier series of the function $f(x) = |x|,\ -\pi < x < \pi$ and hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$. *[(MATH2101.4)(Remember/LOCQ)]*

(b) Find the half range sine series of the function $f(x) = \begin{cases} \frac{1}{4} - x, & 0 < x \le \frac{1}{2} \\ x - \frac{3}{4}, & \frac{1}{2} < x < 1 \end{cases}$.

[(MATH2101.4)(Apply/IOCQ)]

7 + 5 = 12

7. (a) Find the Fourier series of the function $f(x) = \begin{cases} 1, 0 \le x < \frac{\pi}{3} \\ 0,\ \frac{\pi}{3} \le x < \frac{2\pi}{3} \\ -1,\ \frac{2\pi}{3} < x \le \pi \end{cases}$.

[(MATH2101.4)(Understand/LOCQ)]

(b) Obtain the half range cosine series of the function $f(x) = x,\ 0 < x < 2$. Hence, using Parseval's identity deduce that $\frac{\pi^4}{96} = 1 + \frac{1}{3^4} + \frac{1}{5^4} + \cdots$.

[(MATH2101.4)(Evaluate/HOCQ)]

5 + 7 = 12

Group - E

8. (a) The number of monthly breakdown of a computer is a random variable having Poisson distribution with mean equal to 1.8. Find the probability that the computer will function for a month (i) without a breakdown, (ii) with only one breakdown and (iii) with at least one breakdown. *[(MATH2101.3,MATH2101.6)(Apply/IOCQ)]*

(b) If a random variable X follows uniform distribution with parameters a and b, then find (i) mean of X (ii) Var(X). *[(MATH2101.3,MATH2101.6)(Remember/LOCQ)]*

6 + 6 = 12

9. (a) Calculate the mean and standard deviation of the following frequency distribution:

Class	10 – 19	20 – 29	30 – 39	40 – 49	50 – 59	60 – 69
Frequency	3	5	7	9	4	2

[(MATH2101.3, MATH2101.6) (Analyze/IOCQ)]

(b) Out of two regression lines given by $x + 0.2y = 4.2$ and $y + 0.8x = 8.4$, which one is the regression line of "y on x"? Find the mean of x and y. Further find the correlation coefficient between x and y. *[(MATH2101.3,MATH2101.6)(Evaluate/HOCQ)]*

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	37.5	44.79	17.7

Course Outcome (CO):

After the completion of the course students will be able to

MATH2101.1 Apply numerical methods to obtain approximate solutions to mathematical problems where analytic solutions are not possible.

MATH2101.2 Implement appropriate numerical methods for solving advanced engineering problems dealing with interpolation and integration.

MATH2101.3 Design stochastic models to predict the outcomes of events.

MATH2101.4 Recognize the significance of the expansion of a function in Fourier Series.

MATH2101.5 Provide deterministic mathematical solutions to physical problems through partial differential equations.

MATH2101.6 Employ statistical methods to make inferences on results obtained from an experiment.

**LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.*