MATHEMATICS - I
(MTH 1101)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) The condition on t for which the rank of $A = \begin{bmatrix} 2 & 0 & 1 \\ 5 & t & 3 \\ 0 & 3 & 1 \end{bmatrix}$ be 3 is

(a) $t \neq \frac{3}{2}$ (b) $t \neq \frac{2}{3}$ (c) $t = \frac{2}{3}$ (d) $t = \frac{3}{2}$.

(ii) The sequence $\left\{\frac{1}{3^n}\right\}$ is

(a) monotonic increasing (b) divergent
(c) oscillatory (d) monotonic decreasing.

(iii) If 3 is an eigenvalue of the matrix A, then 0 is an eigenvalue of the matrix
(a) A (b) $A - 3I$ (c) $A + 3I$ (d) $A - I$.

(iv) Integrating factor of $\frac{dy}{dx} + y = 1$ is

(a) e^x (b) x^2 (c) x (d) 2.

(v) If $f(x, y, z) = 3x^2y - y^3z^2$, then $grad\ f$ at $(1, -2, -1)$ is
(a) $12\hat{\imath} + 9\hat{\jmath} + 16\hat{k}$ (b) $-12\hat{\imath} - 9\hat{\jmath} - 16\hat{k}$
(c) $\hat{\imath} + 9\hat{\jmath} - 16\hat{k}$ (d) $9\hat{\imath} + \hat{\jmath} - 16\hat{k}$.

(vi) The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + x\left(\frac{dy}{dx}\right)^5 + y = x^2$ is
(a) order 1 degree 5 (b) order 2 degree 5
(c) order 2 degree 3 (d) order 3 degree 5.

(vii) If the differential equation $\left(y + \frac{1}{x} + \frac{1}{x^2y}\right)dx + \left(x - \frac{1}{y} + \frac{A}{xy^2}\right)dy = 0$ is exact, then the value of A is
(a) x (b) 1 (c) $\frac{1}{xy}$ (d) $\frac{1}{y}$.

(viii) The changed order of the integral $\int_0^a \int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} dxdy$ is

(a) $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} dydx$ (b) $\int_0^a \int_0^{\sqrt{2ax-x^2}} dydx$

(c) $\int_0^a \int_0^{-\sqrt{2ax-x^2}} dydx$ (d) $\int_0^{2a} \int_0^{-\sqrt{2ax-x^2}} dydx$.

(ix) If $x = u + v$ and $y = uv$, then $\frac{\partial(x, y)}{\partial(u, v)}$ is

(a) uv (b) $u - v$ (c) $u + v$ (d) $\frac{u}{v}$.

(x) $f(x,y) = \frac{x^2}{y} + \frac{2y^2}{x}$ is a homogeneous function of degree

(a) 0 (b) 1 (c) 2 (d) 3

Fill in the blanks with the correct word

(xi) The eigen values of the matrix $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 3 & -1 \\ 0 & 0 & 5 \end{bmatrix}$ are ________.

(xii) The value of the constant p so that $\vec{f} = (x+3y)\hat{i} + (y-2z)\hat{j} + (x+pz)\hat{k}$ is solenoidal is ________.

(xiii) The determinant of a matrix A, whose eigenvalues be $2, 1, 0$ is ________.

(xiv) The general solution of the differential equation $y - px = \sqrt{p^2 - 1}$, where $p = \frac{dy}{dx}$ is ________.

(xv) If $f(x, y)$ is a homogeneous function of degree n having 2^{nd} order continuous partial derivatives, then $x^2 \frac{\partial^2 f}{\partial x^2} + 2xy \frac{\partial^2 f}{\partial x \partial y} + y^2 \frac{\partial^2 f}{\partial y^2} =$ ________________.

Group - B

2. (a) Reduce the following matrix A in row-reduced echelon form and hence find its rank where $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$. *[(MTH1101.1, MTH1101.2)(Understand/LOCQ)]*

(b) Determine the value of λ & μ for which the system of equations

$x + 2y + 3z = 6$
$x + 3y + 5z = 9$
$2x + 5y + \lambda z = \mu$

will have (i) no solution (ii) a unique solution and (iii) infinitely many solutions.

[(MTH1101.1, MTH1101.2) (Apply/IOCQ)]

6 + 6 = 12

3. (a) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ 2 & 1 & 4 \end{bmatrix}$. Hence compute A^{-1}. *[(MTH1101.1, MTH1101.2)(Remember/LOCQ)]*

(b) Determine the eigenvalues and eigenvectors of the following matrix:
$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 3 & 5 & 1 \end{bmatrix}$. *[(MTH1101.1, MTH1101.2)(Evaluate/HOCQ)]*

6 + 6 = 12

Group - C

4. (a) Show that the sequence $\left\{\frac{2n-1}{3n+2}\right\}$ is monotonic increasing and bounded. Hence prove that it is convergent. *[(MTH1101.3, MTH1101.4)(Analyse/IOCQ)]*

(b) If $r = |\vec{r}|$, where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, prove that
(i) $\vec{\nabla}\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}$ and (ii) $\vec{\nabla}\,(r^n) = n\,r^{n-2}\vec{r}$. *[(MTH1101.3, MTH1101.4)(Understand/LOCQ)]*

6 + 6 = 12

5. (a) Test for absolute convergence or conditional convergence of the series:
$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{3\sqrt{n}}$. *[(MTH1101.3, MTH1101.4)(Remember/LOCQ)]*

(b) Test the convergence of the series $\frac{6}{1.3.5} + \frac{8}{3.5.7} + \frac{10}{5.7.9} + \ldots$.
[(MTH1101.3, MTH1101.4)(Remember/LOCQ)]

(c) Find the value of curl grad ϕ, where $\phi = x^2 + y^2 + z^2$.
[(MTH1101.3, MTH1101.4)(Understand/LOCQ)]

6 + 4 + 2 = 12

Group - D

6. (a) Solve: $(2x\log x - xy)dy + 2y\,dx = 0$. *[(MTH1101.5)(Remember/LOCQ)]*

(b) Solve the following Cauchy-Euler equation:
$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^2\log x$. *[(MTH1101.5)(Evaluate/HOCQ)]*

6 + 6 = 12

7. (a) Solve the following ordinary differential equation:
$p - \frac{1}{p} - \frac{x}{y} + \frac{y}{x} = 0$. *[(MTH1101.5)(Understand/LOCQ)]*

(b) Solve the following ordinary differential equation by the method of variation of parameters: $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = xe^{3x} + sin2x$. *[(MTH1101.5)(Evaluate/HOCQ)]*

6 + 6 = 12

Group - E

8\. (a) Verify Green's theorem, for $\oint_C \{(3x-8y^2)dx+(4y-6xy)dy\}$ where C is the boundary of the region bounded by $x=0,\ y=0$ and $x+y=1.$

[(MTH1101.6)(Apply/IOCQ)]

(b) If $u = f(y-z, z-x, x-y)$, show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z} = 0.$

[(MTH1101.6)(Understand/LOCQ)]

6 + 6 = 12

9\. (a) By using Euler's theorem on homogeneous function prove that

$$x^2\,\frac{\partial^2 u}{\partial x^2} + 2xy\,\frac{\partial^2 u}{\partial x \partial y} + y^2\,\frac{\partial^2 u}{\partial y^2} = \frac{1}{2} cot\, u\left(1 - \frac{1}{2}\, cosec^2\, u\right),$$

where $u = cos^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$.

[(MTH1101.6)(Apply/IOCQ)]

(b) Change the order of the integration and hence evaluate $\int_0^1 \int_{e^x}^{e} \frac{dxdy}{y^2 logy}$.

[(MTH1101.6)(Understand/LOCQ)]

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	56.25	25	18.75

Course Outcome (CO):

After the completion of the course students will be able to

MTH1101.1: Apply the concept of rank of matrices to find the solution of a system of linear simultaneous equations.
MTH1101.2: Develop the concept of eigen values and eigen vectors.
MTH1101.3: Combine the concepts of gradient, curl, divergence, directional derivatives, line integrals, surface integrals and volume integrals.
MTH1101.4: Analyze the nature of sequence and infinite series. .
MTH1101.5: Choose proper method for finding solution of a specific differential equation.
MTH1101.6: Describe the concept of differentiation and integration for functions of several variables with their applications in vector calculus.

**LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.*