FUNDAMENTALS OF DIGITAL SIGNAL PROCESSING
(AEIE 3104)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) The sampling frequency of the analog signal, $x(t) = 4\sin 150\pi t + 2\cos 50\pi t$ should be,
(a) greater than 75 Hz (b) greater than 150 Hz
(c) less than 150 Hz (d) greater than 50 Hz.

(ii) In a signal $x(n)$, if 'n' is replaced by $\frac{n}{3}$, then it is called
(a) up sampling (b) folded version
(c) down sampling (d) shifted version.

(iii) The ROC of the signal $x(n) = a^n$ for $-5 < n < 5$ is
(a) entire z-plane
(b) entire z-plane except $z = 0$ and $z = \infty$
(c) entire z-plane except $z = 0$
(d) convolution.

(iv) The Z-transform of $a^{-n}u(-n-1)$ is
(a) $\frac{-z}{z-\frac{1}{a}}$ (b) $\frac{z}{z-\frac{1}{a}}$
(c) $\frac{z}{z-a}$ (d) $\frac{-z}{z-a}$

(v) The value of the twiddle factor W_8^4 is given by
(a) 1 (b) $-j$
(c) $\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$ (d) -1

(vi) The complex valued phase factor/ twiddle factor, W_N can be represented as
(a) $e^{-j\,2\pi N}$ (b) $e^{-j\frac{2\pi}{N}}$
(c) $e^{-j\,2\pi}$ (d) $e^{-j\,2\pi kN}$

(vii) For the analog and digital IIR filters to be causal, the number of zeros should be
(a) ≥ Number of poles (b) ≤ Number of poles
(c) = Number of poles (d) Zero.

(viii) The unnormalized transfer function of lowpass Butterworth filter is obtained from normalized transfer function by replacing s_n by,

(a) $\frac{s_n}{\Omega_c}$ (b) $s_n\Omega_c$

(c) $\frac{s}{\Omega_c}$ (d) $s\Omega_c$

(ix) The Characteristics of ideal linear phase FIR filter are

(a) $|H(e^{j\omega})|$ = Constant and $\angle H(e^{j\omega}) = \frac{1}{\omega}$

(b) $|H(e^{j\omega})|$= Constant and $\angle H(e^{j\omega}) = -\alpha\omega$

(c) $|H(e^{j\omega})| = -\alpha\omega$ and $\angle H(e^{j\omega})$ =Constant

(d) $|H(e^{j\omega})| = \frac{1}{\omega}$ and $\angle H(e^{j\omega})$ =Constant.

(x) The zeros of the Butterworth filters exist at

(a) left half of s-plane (b) origin

(c) infinity (d) right half of s-plane.

Fill in the blanks with the correct word

(xi) The correlation of two different discrete time sequences is called _____.

(xii) The tolerance in the pass band and stop band are called _____.

(xiii) The phenomena of high frequency components acquiring the identity of low frequency components is called _____.

(xiv) Appending zeros to a sequence in order to increase its length is called _____.

(xv) An LTI system is causal if and only if its impulse response is _____ for negative values of n.

Group - B

2. (a) What are energy and power signals? *[(CO1)(Understand/LOCQ)]*

(b) Determine whether the signal $x(n) = e^{\frac{j7\pi n}{4}}$ is periodic or not. If periodic find the fundamental period? *[(CO1)(Apply/IOCQ)]*

(c) Find the z-transform of the following discrete time signal and find its ROC $x(n) = (n + 0.5)\left(\frac{1}{3}\right)^n u(n)$. *[(CO2)(Apply/IOCQ)]*

(d) Prove that, if $X(z) = Z\{x(n)\}$, then $Z\{x(n-m)\} = z^{-m}X(z)$. *[(CO2)(Analyse/IOCQ)]*

2 + 3 + 4 + 3 = 12

3. (a) Determine whether the signal $x(n) = \left(\frac{1}{4}\right)^n u(n)$ is energy or power signal. *[(CO1)(Apply/IOCQ)]*

(b) Prove that if $X(z) = Z\{x(n)\}$, then $Z\{nx(n)\} = -z\frac{d}{dz}X(z)$. *[(CO2)(Analyse /IOCQ)]*

(c) Determine the causal signal $x(n)$ having the z-transform $X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$. *[(CO2) (Apply/IOCQ)]*

4 + 4 + 4 = 12

Group - C

4. (a) Compute the DFT of the sequence, $x(n) = \{0, 1, 2, 1\}$. *[(CO3)(Apply/IOCQ)]*
   (b) Prove, if $X_3(k) = X_1(k)X_2(k)$ then $x_3(n) = \sum_{m=0}^{N-1} x_1(m)x_2\big((n-m)\big)_N$. *[(CO3)(Analyse/IOCQ)]*

8 + 4 = 12

5. (a) Compute the circular convolution of two finite duration sequences $x_1(n) = \{1, -1, -2, 3, -1\}$ and $x_2(n) = \{1, 2, 3\}$. *[(CO3)(Evaluate/HOCQ)]*
   (b) Find the output $y(n)$ of a filter whose impulse response is $h(n) = \{1, 1, 1\}$ and input signal $x(n) = \{3, -1, 0, 1, 3, 2, 0, 1, 2, 1\}$ using overlap-save method. *[(CO3)(Apply/IOCQ)]*

6 + 6 = 12

Group - D

6. (a) Design an analog Butterworth filter that has a $-2\ dB$ passband attenuation at a frequency of $20\ rad/sec$ and at least $-10\ dB$ stopband attenuation at $30\ rad/sec$. *[(CO4,CO5)(Evaluate/HOCQ)]*
   (b) Realize the system given by difference equation $y(n) = -0.1y(n-1) + 0.2y(n-2) + 3x(n) + 3.6x(n-1) + 0.6x(n-2)$ in direct form I. *[(CO5)(Analyse/IOCQ)]*
   (c) Differentiate between FIR and IIR filters. *[(CO4)(Understand/LOCQ)]*

6 + 4 + 2 = 12

7. (a) State some advantages of digital filters. *[(CO4)(Understand/LOCQ)]*
   (b) What are the requirements for a digital filter to be stable and causal? *[(CO4)(Remember/LOCQ)]*
   (c) The frequency response of a digital filter is, $H\big(e^{jw}\big) = (0.7 + 0.6\cos\omega - 0.9\cos 2\omega e - j7.5\omega$. Determine the phase delay and group delay. *[(CO5)(Apply/IOCQ)]*
   (d) What are the possible types of impulse response for Linear phase FIR filters? *[(CO4)(Remember/LOCQ)]*

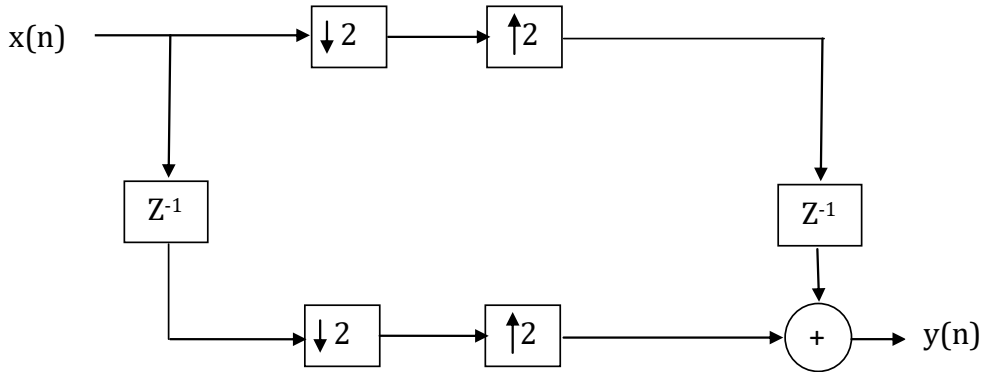
2 + 3 + 4 + 3 = 12

Group - E

8. (a) What is the difference between DFT and DCT? Write down few applications of DCT. *[(CO6)(Remember/LOCQ)]*
   (b) What is Wavelet Transform? What are the applications of wavelets? Write down the expression of forward and inverse continuous wavelet transform. Write down their applications in signal processing. *[(CO6)(Understand/LOCQ)]*

(2 + 2) + (2 + 2 + 2 + 2) = 12

9. A multirate systems is shown below. Find the relation between $x(n)$ and $y(n)$.



[(CO6)(Analyse/HOCQ)]

(b) What is decimation and interpolation? *[(CO6)(Remember/LOCQ)]*

(c) What is anti-aliasing filter? *[(CO6)(Remember/LOCQ)]*

6 + 4 + 2 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	31.25	50	18.75

Course Outcome (CO):

After the completion of the course students will be able to

1. Characterize and analyze the properties of discrete time signals and systems.
2. Analyze a discrete linear time invariant system using Z-transform.
3. Perform Fourier Transform of Discrete-Time signals and learn implementation of Fast Fourier Transform algorithms.
4. Distinguish between analog and digital filter, methods to transform from one type to another types of filter.
5. Design digital FIR and IIR filters according to the given specification and realize structure of a digital filter for given transfer function
6. Familiarize with short time Fourier transform, discrete cosine transform, wavelet transform and multirate digital signal processing

**LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.*