MATHEMATICAL METHODS
(MATH 2001)

Time Allotted : 2½ hrs **Full Marks : 60**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 4 (four) from Group B to E, taking one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A

1. Answer any twelve: **12 × 1 = 12**

Choose the correct alternative for the following

(i) The singular point of the ordinary differential equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 4y = 0$
(a) is 0 (b) is 1 (c) does not exist (d) is -1.

(ii) The partial differential equation $\frac{\partial^2 z}{\partial x^2} = \left(\frac{\partial z}{\partial y}\right)^{\frac{1}{3}} + 3$ is of
(a) order 3, degree 2 (b) order 2, degree 3
(c) order 2, degree 2 (d) order 3, degree 3.

(iii) A function which satisfies Dirichlet's condition can be represented as an infinite series of
(a) algebraic functions (b) sinusoidal functions
(c) exponential functions (d) logarithmic functions.

(iv) $\frac{d}{dx}\{x^4 J_4(x)\} =$
(a) $x^4 J_5(x)$ (b) $x^4 J_3(x)$
(c) $x^3 J_4(x)$ (d) $-x^3 J_4(x)$.

(v) If $f(z) = \frac{z-3}{z(z+1)}$, then the point $z = 0$ is
(a) a removable singularity (b) an isolated singularity
(c) an essential singularity (d) a simple pole.

(vi) Conjugate harmonic of $e^x \cos y$ is
(a) $e^{-x} \cos y$ (b) $-e^x \sin y$
(c) $e^x \sin y$ (d) $e^{-x} \sin y$.

(vii) If $F(s)$ be the Fourier transform of $f(t)$, then Fourier transform of $tf(t)$ is
(a) $\frac{d}{ds}\{F(s)\}$ (b) $i\frac{d}{ds}\{F(s)\}$
(c) $-i\frac{d}{ds}\{F(s)\}$ (d) $\frac{d}{ds}\{sF(s)\}$.

(viii) Bessel's equation of order zero is

(a) $xy'' + y' - xy = 0$ (b) $xy'' + y' = 0$

(c) $xy'' - y' + xy = 0$ (d) $xy'' + y' + xy = 0$.

(ix) If $3y - 5x^2 + my^2$ is a harmonic function, then the value of m is

(a) 2 (b) 3 (c) 5 (d) 0.

(x) The solution of the partial differential equation $z = px + qy + \frac{1}{p+q}$ is (where a, b are arbitrary constants, $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$)

(a) $z = ax + by + \frac{1}{a+b}$ (b) $z = ax + by + \frac{1}{a-b}$

(c) $z = ax + by - ab$ (d) $z = ax - by + \frac{1}{a+b}$.

Fill in the blanks with the correct word

(xi) If $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$ is the Fourier series of $f(x) = \frac{1}{4}(\pi - x)^2$, then a_0 is ___________.

(xii) One dimensional Heat equation is _________.

(xiii) If C be the circle $|z| = 1$, then $\int_C \frac{\cos 2z}{z - \frac{\pi}{6}} dz$ is _________.

(xiv) The value of $J_{\frac{1}{2}}^2(x) + J_{-\frac{1}{2}}^2(x)$ is _________.

(xv) The particular integral of $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = \sin(x + y)$ is _________.

Group - B

2. (a) Show that the function $f(z) = \begin{cases} \frac{2xy(x+iy)}{x^2+y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ satisfies the Cauchy-Riemann equations at $z = 0$, although $f(z)$ is not analytic at the origin.

[(MATH2001.1, MATH2001.2)(Analyse/IOCQ)]

(b) Evaluate: $\int_C \frac{z-1}{(z+1)^2(z-2)} dz$ on the circle $C: |z - i| = 3$.

[(MATH2001.1,MATH2001.2)(Evaluate/HOCQ)]

6 + 6 = 12

3. (a) Expand the function $f(z) = \frac{\sin z}{z}$ in Laurent's series about the point $z = 0$. Hence classify the type of the singular points of $f(z)$.

[(MATH2001.1, MATH2001.2)(Understand/LOCQ)]

(b) Use Cauchy's residue theorem to evaluate the integral $\oint_C \frac{e^z-1}{z(z-1)(z-i)^2} dz$, where C is the circle $|z| = 2$.

[(MATH2001.1, MATH2001.2) (Apply/IOCQ)]

6 + 6 = 12

Group - C

4. (a) Find the Fourier series of the following function in $[-\pi, \pi]$.

$$f(x) = \begin{cases} 0, & \text{for } -\pi < x \leq 0, \\ \dfrac{\pi x}{4}, & \text{for } 0 < x < \pi. \end{cases}$$

How $f(x)$ would be defined at $x = 0$ so that the series converges to $f(x)$.

[(MATH2001.1, MATH2001.3, MATH2001.4)(Evaluate/HOCQ)]

(b) Evaluate the inverse Fourier transform of $f(s) = \frac{1}{s^2+4s+13}$.

[(MATH2001.1, MATH2001.3, MATH2001.4)(Remember/LOCQ)]

6 + 6 = 12

5. (a) Find half range cosine series of $f(x) = \begin{cases} \pi x, & 0 < x < 1 \\ \pi(2-x), & 1 \leq x < 2 \end{cases}$.

Hence using Parseval's identity show that $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots$

[(MATH2001.1, MATH2001.3, MATH2001.4) (Apply/IOCQ)]

(b) Find the Fourier transform of

$$f(x) = \begin{cases} x^2, & \text{for } |x| < a, \\ 0, & \text{for } |x| \geq a. \end{cases}$$

[(MATH2001.1, MATH2001.3, MATH2001.4) (Apply/IOCQ)]

6 + 6 = 12

Group - D

6. (a) Find the series solution of the following ordinary differential equation:

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + x^2y = 0, \text{ about } x = 0.$$

[(MATH2001.5)(Evaluate/HOCQ)]

(b) Express $J_4(x)$ in terms of $J_0(x)$ and $J_1(x)$. *[(MATH2001.5)(Remember/LOCQ)]*

8 + 4 = 12

7. (a) Define the Bessel's function of 1st kind of order n and hence prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$. *[(MATH2001.5)(Apply/IOCQ)]*

(b) Prove that $(2n+1)x\, P_n(x) = (n+1)\, P_{n+1}(x) + n\, P_{n-1}(x)$, where $P_n(x)$ denotes the Legendre polynomial of degree n. *[(MATH2001.5)(Apply/IOCQ)]*

6 + 6 = 12

Group - E

8. (a) Form a partial differential equation by eliminating arbitrary function ϕ from the relation $\phi(x+y+z, x^2+y^2+z^2) = 0$. *[(MATH2001.1,MATH2001.6)(Understand/LOCQ)]*

(b) Use Charpit's method to find a complete integral of the partial differential equation $2xz - px^2 - 2qxy + pq = 0$, where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$.

[(MATH2001. 1,MATH2001. 6)(Apply/IOCQ)]

6 + 6 = 12

9. (a) Solve the partial differential equation:

$x(y^2 - z^2)p - y(z^2 + x^2)q = z(x^2 + y^2)$, where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$.

[(MATH2001. 1,MATH2001. 6)(Apply/IOCQ)]

(b) Solve: $(D^2 - DD' - 2D'^2)z = 2x + 3y$, where $D \equiv \frac{\partial}{\partial x}$ and $D' \equiv \frac{\partial}{\partial y}$.

[(MATH2001.1, MATH2001.6)(Remember/LOCQ)]

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	29.17	50	20.83

Course Outcome (CO):

After the completion of the course students will be able to

MTH1101.1 Apply the concept of rank of matrices to find the solution of a system of linear simultaneous equations.
MTH1101.2 Develop the concept of eigen values and eigen vectors.
MTH1101.3 Combine the concepts of gradient, curl, divergence, directional derivatives, line integrals, surface integrals and volume integrals.
MTH1101.4 Analyze the nature of sequence and infinite series
MTH1101.5 Choose proper method for finding solution of a specific differential equation.
MTH1101.6 Describe the concept of differentiation and integration for functions of several variables with their applications in vector calculus.

**LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.*