OPERATIONS RESEARCH
(MATH 2203)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) If a function is convex on a convex domain then any local minimum is
(a) global maximum (b) global minimum
(c) local maximum (d) neither global maximum nor global minimum.

(ii) Let $Q(x,y,z)$ be a quadratic form such that $Q(1,1,1) = 10$ and $Q(-1,-2,3) = -25$, then
(a) $Q(x,y,z)$ could be indefinite
(b) $Q(x,y,z)$ could be positive semi definite
(c) $Q(x,y,z)$ could be negative definite
(d) $Q(x,y,z)$ could be negative semi definite.

(iii) The optimal solution of the following game problem is

PLAYER B

PLAYER A		B_1	B_2	B_3	B_4
	A_1	1	7	3	4
	A_2	5	6	4	5
	A_3	7	2	0	3

(a) (A_3,B_2) (b) (A_2,B_2) (c) (A_3,B_4) (d) (A_2,B_3)

(iv) Which of the following Hessian matrices belongs to a concave function?
(a) $\begin{pmatrix} 2 & x \\ 0 & -x^2 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 2 \\ 1 & x^2 \end{pmatrix}$ (c) $\begin{pmatrix} -x^2 & x \\ 0 & x \end{pmatrix}$ (d) $\begin{pmatrix} -2 & x \\ -x & -1 \end{pmatrix}$.

(v) Which of the following statements is TRUE?
(a) In a two person zero sum game gains of one player are equal to the losses of other player.
(b) Every matrix game has a saddle point.
(c) The concept of dominance in reducing the size of a matrix game may lead to the loss of the saddle point.
(d) If a game has a saddle point, then the players play with their mixed strategies.

(vi) The local minimum of the function $f(x)$ occurs at $x = x_0$ provided
(a) $f^{(n)}(x_0) < 0$, for n odd (b) $f^{(n)}(x_0) > 0$, for n odd
(c) $f^{(n)}(x_0) > 0$, for n even (d) $f^{(n)}(x_0) < 0$,for n even.
(where $f^{(n)}(x_0)$ denotes the n^{th} order derivative of the function $f(x)$at $x = x_0$)

(vii) If the primal and the dual problem have feasible solutions, then
(a) dual objective function is unbounded
(b) finite optimal for both exists
(c) primal objective function is unbounded
(d) finite optimal for both do not exist.

(viii) The MODI method in Transportation problem is used to find the
(a) optimum solution (b) basic solution
(c) non basic solution (d) trivial solution.

(ix) For a maximization model in an LPP, the coefficient of an artificial variable is
(a) 1 (b) 0 (c) M (d) $-M$.

(x) The value of golden ratio is
(a) 0.618 (b) -0.618 (c) 1.618^2 (d) 1.618.

Group - B

2\. (a) Find the graphical solution of the given L.P.P:
Maximize $z = 2x_1 + 5x_2$
subject to the constraints

$$x_1 + 2x_2 \leq 2$$
$$2x_1 + 3x_2 \geq 6$$
$$x_1, \quad x_2 \geq 0$$

[(MATH2203.1, MATH2203.2)(Understand/LOCQ)]

(b) Solve the following L.P.P by simplex algorithm:
Maximize $z = 7x_1 + 14x_2$
subject to the constraints

$$3x_1 + 2x_2 \leq 36$$
$$x_1 + 4x_2 \leq 10$$
$$x_1, \ x_2 \geq 0$$

[(MATH2203.1, MATH2203.2)(Apply/IOCQ)]

5 + 7 = 12

3\. (a) Use Big-M method to solve the following L.P.P.:
Maximize $z = 2x_1 + 3x_2$
subject to the constraints

$$x_1 + x_2 \geq 2$$
$$x_1 + 2x_2 \leq 8$$
$$x_1, \quad x_2 \geq 0.$$

[(MATH2203.1, MATH2203.2)(Apply/IOCQ)]

(b) Find the dual of the given primal
Minimize $z = 10x_1 + 8x_2$
subject to the constraints

$$x_1 + 2x_2 \geq 5$$
$$2x_1 - x_2 \geq 12$$

$$x_1 + 3x_2 \geq 4$$
$$x_1 \geq 0 \text{ and } x_2 \text{ is unrestricted.}$$

[(MATH2203.1, MATH2203.2)(Understand/LOCQ)]

7 + 5 = 12

Group - C

4. (a) Solve the given transportation problem using VAM and hence find its optimal solution.

	I	II	III	Supply
A	5	1	7	10
B	6	4	6	80
C	3	2	5	15
D	5	3	2	40
Demand	75	20	50	

[(MATH2203.1, MATH2203.2, MATH2203.3, MATH2203.4)(Evaluate/HOCQ)]

(b) Solve the assignment problem where the cost of assigning jobs J_1 to J_4 to workers W_1 to W_4 is given below:

	J_1	J_2	J_3	J_4
W_1	10	15	24	30
W_2	16	20	28	10
W_3	12	18	30	16
W_4	9	24	32	18

[(MATH2203.1, MATH2203.2, MATH2203.3, MATH2203.4)(Evaluate/HOCQ)]

7 + 5 = 12

5. (a) Use dominance to reduce the following pay-off matrix to a 2×2 game and hence find the optimal strategies and the value of the game:

PLAYER B

PLAYER A				
	3	2	4	0
	3	4	2	4
	4	2	4	0
	0	4	0	8

[(MATH2203.1, MATH2203.2, MATH2203.3, MATH2203.4)(Apply/IOCQ)]

(b) Use graphical method in solving the following game and find the value of the game:

PLAYER B

PLAYER A		
	−6	7
	4	−5
	−1	−2
	−2	5
	7	−6

[(MATH2203.1, MATH2203.2, MATH2203.3, MATH2203.4)(Understand/LOCQ)]

6 + 6 = 12

Group - D

6. (a) Use Kuhn-Tucker conditions to solve the following non-linear programming problem:
Maximize $z = 2x_1 + 3x_2 - x_1^2 - 2x_2^2$
subject to the constraints

$$x_1 + 3x_2 \le 6$$
$$5x_1 + 2x_2 \le 10$$
$$x_1, x_2 \ge 0.$$

[(MATH2203.6)(Apply/IOCQ)]

(b) Determine the relative maximum and minimum (if any) of the following function:
$f(x_1, x_2, x_3) = x_1 + x_1x_2 + 2x_2 + 3x_3 - x_1^2 - 2x_2^2 - x_3^2$.

[(MATH2203.6)(Understand/LOCQ)]

8 + 4 = 12

7. Use the method of Lagrangian multipliers to solve the following non- linear programming problem. Does the solution maximize or minimize the objective function?

$$\text{Optimize } Z = 2x_1^2 - 24x_1 + 2x_2^2 - 8x_2 + 2x_3^2 - 12x_3 + 200$$

Subject to the constraint

$$x_1 + x_2 + x_3 = 11$$
$$x_1, x_2, x_3 \ge 0.$$

[(MATH2203.6)(Evaluate/HOCQ)]

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Group - E

8. Use Interval Halving method to minimize $f(x) = x^4 - 12x^2 - 60x$ over [0,2] taking the tolerance to be less than 0.3. *[(MATH4121.1, MATH4121.5)(Apply/IOCQ)]*

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9. Use Golden Section search algorithm to minimize $f(x) = x^2(x - 2.5)$ in $[0, 1]$ taking the stopping tolerance to be less than 0.25. *[(MATH2203.1, MATH2203.5)(Apply/IOCQ)]*

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Cognition Level	*LOCQ*	*IOCQ*	*HOCQ*
Percentage distribution	*20.83*	*54.17*	*25*

Course Outcome (CO):

After the completion of the course students will be able to

MATH2203.1 Describe the way of writing mathematical model for real-world optimization problems.
MATH2203.2 Identify Linear Programming Problems and their solution techniques.
MATH2203.3 Categorize Transportation and Assignment problems.
MATH2203.4 Apply the way in which Game theoretic models can be useful to a variety of real-world scenarios in economics and in other areas.
MATH2203.5 Apply various optimization methods for solving realistic engineering problems and compare their accuracy and efficiency.
MATH2203.6 Convert practical situations into non-linear programming problems and solve unconstrained and constrained programming problems using analytical techniques.

**LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.*