THEORY OF COMPUTATION
(CSEN 5234)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) A minimum state DFA accepting the language L = {w | w ∈ (0,1)*, where, the number of 0's and 1's of w are divisible by 5 and 4, respectively} has
(a) 10 states (b) 20 states (c) 15 states (d) 16 states

(ii) The Pumping Lemma for Regular Languages can be used to prove that a given language
(a) is Type 2 (b) is Type 3
(c) is not Type 2 (d) is not Type 3.

(iii) Let S & T be language over ∑: {a, b} represented by the following regular expressions (a + b*)* and (a + b)* respectively, then which of the following is TRUE?
(a) S ⊂ T (b) T ⊂ S (c) S = T (d) S ∩ T = Ø.

(iv) Consider the following three languages:
$L_1 = \{ww^R \mid w \in (0,1)^*\}$
$L_2 = \{w \# w^R \mid w \in (0,1)^*\}$, where # is a special symbol
$L_3 = \{ww \mid w \in (0,1)^*\}$
Which one of the following is false?
(a) L1 is a deterministic CFL (b) L2 is a deterministic CFL
(c) L3 is a CFL (d) both (a) and (c).

(v) Suppose that L_4 and L_5 are two languages (over the same alphabet) given to you such that both L_4 and $L_4 \cap L_5$ are CFL. Then which of the following is incorrect?
(a) L_5 must be CFL too (b) L_5 can never be regular
(c) L_5 can be regular (d) Both (a) & (b).

(vi) If the original nondeterministic finite state automata (ndfsa) has n states, the corresponding deterministic finite state automata (dfsa) cannot have
(a) more than n states (b) more than 2n states
(c) more than 2^n states (d) more than (n-1) states.

(vii) Which one of the following languages cannot be accepted by a non-deterministic pushdown acceptor (ndpda)?
(a) $\{ 0^m1^n \mid 0 < m, 0 < n \}$ (b) $\{ 0^m1^n0^m \mid 0 < m, 0 < n \}$
(c) $\{ 0^m1^m0^m \mid 0 < m \}$ (d) $\{ 0^m1^n0^r \mid 0 < m, 0 < n, 0 < r \}$.

(viii) If a Context Free Grammar is in Chomsky Normal Form then a production
(a) may contain a single non-terminal in the RHS
(b) may contain both terminal and non-terminal symbols in the RHS
(c) may be of the form $A \rightarrow \varepsilon$ where A is not the start symbol
(d) none of these.

(ix) If a Context Free Language L_3 has a Type 2 grammar G which is ambiguous then there must be a terminal string α such that
(a) α has two different derivation trees
(b) α has two different rightmost derivations
(c) α has two different leftmost derivations
(d) all of the above.

(x) Let L_6 be a recursive language, and L_7 be a recursively enumerable language but not recursive language. Which of the following is true?
(a) L_6' is recursive L_7' is recursively enumerable
(b) L_6' is recursive L_7' is not recursively enumerable
(c) L_6' andL_7' are recursively enumerable
(d) L_6' is recursively enumerable L_7' is recursive.

Group – B

3. (a) A non-deterministic finite state acceptor (ndfsa) M_2 has the state table shown below. S is the start state, and C is the only goal state. Convert M_2 to an equivalent deterministic finite state acceptor (dfsa) M_3 and clearly indicating the start and goal states.

	0	**1**
→S	**P**	**Q**
P	**P, A**	**B,C**
Q	**Q, A**	**B,C**
A	**A,C**	**----**
B	**---**	**C**
***C**	**C**	**C**

[(CO1)(Apply/IOCQ)]

(b) Construct an DFA over alphabet {0, 1} that accepts those strings that contain the pattern 001 somewhere. *[(CO2)(Create/HOCQ)]*

(c) What do you mean by 2- equivalent states? *[(CO1)(Remember/LOCQ)]*

5 + 5 + 2 = 12

2. (a) Design a DFA that accepts every string over $\Sigma = \{a, b\}$ which starts and ends in same symbol. *[(CO1)(Create/IOCQ)]*

(b) Consider the DFA given below (q1 is the starting state and q3, q5 are two final states)

δ	0	1
q_1	q_2	q_3
q_2	q_3	q_5
q_3	q_4	q_3
q_4	q_3	q_5
q_5	q_2	q_5

Find out the indistinguishable and distinguishable states for the automata.
Construct minimum state equivalent of automata. *[(CO1)(Apply/LOCQ)]*

6 + 6 =12

Group – C

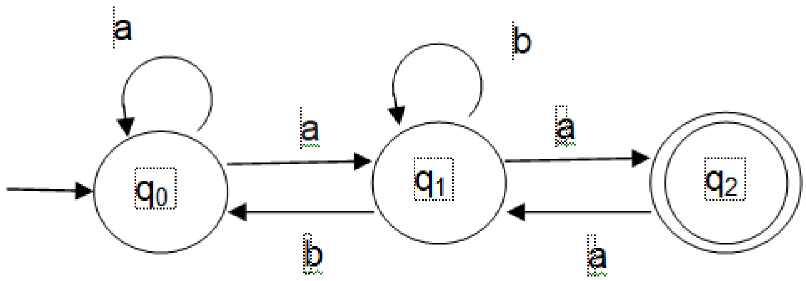
4. (a) Consider the language $L_8 = \{x \in \{0, 1\}^* \mid x$ does not start with 010}. Show that the language is regular. *[(CO2)(Apply/IOCQ)]*

(b) Let L_{10} be the set of all palindromes over {a, b}.
Construct a grammar G generating L_{10}.
According to Chomsky classification, which type of grammar is this?
Can we construct a nondeterministic finite automaton for this language? Justify your answer. *[(CO6)(Create/HOCQ)]*

5 + (3 + 2 + 2) = 12

5. (a) Prove that $(1 + 00^*1) + (1 + 00^*1)(0 + 10^*1)^* (0 + 10^*1) = 0^*1(0 + 10^*1)^*$ *[(CO2)(Apply/IOCQ)]*

(b) Consider the transition system given in the figure, below.
Prove that the strings recognized are given by $(a + a(b + aa)^*b)^* a(b + aa)^* a$. *[(CO2)(Apply/IOCQ)]*



(c) What do you mean by right linear grammar? Give an example. *[(CO6)(Understand/LOCQ)]*

3 + 6 + 3 = 12

Group – D

6. (a) Simplify the given CFG.

$S \rightarrow ACD$
$A \rightarrow a \mid F$
$B \rightarrow \varepsilon$
$C \rightarrow ED \mid \varepsilon$
$D \rightarrow BC \mid b$
$E \rightarrow b$
$F \rightarrow aF$

[(CO3)(Create/HOCQ)]

(b) Consider the CFG given below:

$S \rightarrow a \mid Sa \mid bSS \mid SSb \mid SbS$

Is the CFG ambiguous? Justify your answer. *[(CO3)(Apply/IOCQ)]*

6 + 6 = 12

7. (a) Design a pushdown automata accepting L= {w ∈ {a,b}* : number of a's in w is exactly double of the number of b's in w}. *[(CO4)(Create/HOCQ)]*

(b) Using the Pumping Lemma for Context-Free Languages, show that the language $L = \{ 0^a1^b0^a1^b \mid a > 0, b > 0 \}$ is not a Type 2 (context-free) language. *[(CO3)(Evaluate/HOCQ)]*

6 + 6 = 12

Group – E

8. (a) Nobody knows yet if P = NP. Consider the language L_{11} defined as follows:

$$L_{11} = \begin{cases} (0+1)^* & \text{if } P = NP, \\ \emptyset & \text{otherwise} \end{cases}$$

Is L_{11} recursive or recursively enumerable? Justify your answer. *[(CO7)(Analyse/IOCQ)]*

(b) A single tape Turing machine (M) has three states q0, q1, and q2, where, q0 is the starting state. The tape symbols of M are {a, b, B} and I/P alphabets are {a, b}. The symbol B is the blank symbol. The transition function of the M is given below:

	a	b	B
q0	q1, a, R	-	q0, B, R
q1	-	q2, b, R	-
q2	q2, a, R	q2, b, R	Halt

What is the language accepted by the Turing machine M? *[(CO5)(Apply/LOCQ)]*

(c) Write a short note on multi-tape Turing machine. *[(CO5)(Remember/LOCQ)]*

4 + 4 + 4 = 12

9. (a) Design a Turing machine that can compute the sum of two numbers. *[(CO5)(Apply/IOCQ)]*

(b) Design a Turing machine M_{11} that find the 1's complement of a binary number. e.g. if the input number is 00110 then the output will be 11001. *[(CO5)(Create/HOCQ)]*

6 + 6 = 12

Cognition Level	*LOCQ*	*IOCQ*	*HOCQ*
Percentage distribution	*26.04*	*36.46*	*37.50*

Course Outcome (CO):

After the completion of the course students will be able to

1. Design and analyze Deterministic and non-deterministic finite state automata.
2. Understand the correspondence between finite state automata and regular languages.
3. Design context free grammars to generate strings from a context free language and convert them into Chomsky normal forms.
4. Design deterministic and non-deterministic push down automata to recognize context free languages.
5. Construct Touring machines for computable functions.
6. Understand the hierarchy of formal languages, grammars and machines.
7. Distinguish between computability and non-computability and Decidability and undecidability.

**LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question*