ADVANCED ALGORITHMS
(CSEN 5201)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) Which of the following functions is asymptotically largest?
(a) 2^n (b) $n^{\log n}$ (c) $n^{\sqrt{n}}$ (d) $\sqrt[3]{\log n}$.

(ii) Let G = (V,E) be a weighted directed graph with n **>** 0 vertices and e **>** 0 edges. Let s be the start vertex in V. All vertices are reachable from s, and edge weights are positive integers. Then the time taken by Dijkstra's algorithm to find the shortest paths from s to all other vertices in V is
(a) O(n) (b) O(n lg n) (c) O(n lg e) (d) O(e lg n).

(iii) G = (V,E) is a connected undirected graph. It is known that there is a cycle in G. If V contains 25 nodes, then the number of edges in E is not less than
(a) 22 (b) 23 (c) 24 (d) 25.

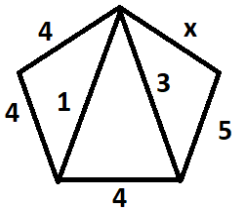
(iv) Which one is true of the following?
(a) All NP-Hard problems are NP-Complete
(b) All NP-Complete problems are NP-Hard
(c) Some NP-Complete problems are NP-Hard
(d) None of these.

(v) When an array containing n positive integers is converted into a max-heap, the height of the max-heap (viewed as a tree structure) is
(a) $\lfloor \lg n \rfloor$ (b) $\lceil \lg n \rceil$ (c) n (d) n-1.

(vi) What is the time complexity to perform an EXTRACT_MIN in a MIN-Heap of size n?
(a) O(n) (b) $O(n^2)$ (c) O(logn) (d) $O(\sqrt{n})$.

(vii) The recurrence relation T(n) **=** 2.T(n-1) **+** 2, n **>** 1, given the initial condition T(1) **=** 0, has the solution (for n >= 1)
(a) T(n) = 3n (b) $T(n) = 2^n$ (c) $T(n) = 2^n - 2$ (d) $T(n) = n^2 - 1$.

(viii) Consider the undirected connected graph given below:



What will be the value of x, such that it will maximize the number of MSTs?
(a) 5 (b) 4 (c) 1 (d) 3.

(ix) Which one of the following problems, in its commonly used formulation, does not currently have a polynomial-time algorithm in the size of the input?
(a) The Fractional Knapsack Problem
(b) The Travelling Salesman Problem
(c) The All Pairs Shortest Path Problem
(d) The Minimum-Cost Spanning Tree Problem.

(x) Let A[1: n] be an array storing 1 or 0 at each location and fun(m) is a function, which has a time complexity Θ(m). Now consider the program segment:

```
count = 0;
for(i  = 1; i <= n; i++)
{
        if(A[i] == 1) count++;
        else
        {
            fun(counter);
            counter = 0;
    }
}
```

The worst-case time complexity of the program fragment is
(a) O(n) (b) $O(n^2 \log n)$ (c) $O(n^2)$ (d) O(nlogn).

Group- B

2. (a) Prove that MAX-Heap of size n can be built in O(n) time. *[(CO3,CO4)(Analyse/IOCQ)]*
   (b) Write a recursive algorithm to find out the Fibonacci numbers and analyse the time complexity of your algorithm. *[(CO4)(Apply/IOCQ)]*

6 + 6 = 12

3. (a) Does there exist any comparison-based sorting algorithm that is asymptotically *not lower bounded* by ($n \log_2 n$)? Justify your answer.
[(CO1, CO4, CO5)(Understand, Remember/LOCQ)]

   (b) Convert the array [3, 6, 8, 4, 9, 2, 5, 7] into a max-heap. How many exchanges are needed in the array to convert it into a max-heap? Justify your answer.
[(CO2)(Understand/IOCQ)]

(3 + 3) + (4 + 2) = 12

Group - C

4. (a) Let G = (V,E) is a directed graph with 15 vertices numbered 1 through 15 and the following 22 directed edges: (1,2), (1,3), (1,4), (2,8), (3,9), (4,3), (4,5), (5,6), (6,11), (6,12), (7,13), (8,7), (8,13), (8,14), (9,10), (9,14), (10,14), (10,15), (11,15), (12,11), (13,15), (14,15). Each edge is directed from the first to the second vertex.
Traverse the graph using Depth-first search. Display the order in which nodes are traversed. Briefly outline the steps followed. *[(CO3](Apply/LOCQ)]*

(b) How to determine the topological sorting order of a directed graph using Depth-first search? Find out the topological sorting order of the graph given in 4(a).
[(CO2)(Remember/HOCQ)]
8 + 4 = 12

5. (a) Give the pseudo-code for Kruskal's algorithm for MST with a very brief explanation of how it works. Note that you do NOT need to write the implementation details of disjoint-set data structure. Then do a complexity analysis for Kruskal algorithm. *[(CO2,CO4,CO5)(Remember,Understand/LOCQ)]*

(b) Apply your algorithm on the graph G stated below, to produce the MST of the graph G, where G = (V,E) is a weighted undirected graph with 5 vertices and 8 edges. The vertices are numbered 1 through 5, and the edges are as follows: (1,2,8), (1,4,5), (1,5,3), (2,3,3), (2,5,-7), (4,3,2), (3,5,6), (4,5,5). Here (u,v,w) represents two end vertics of an edge, an w is the weight of that edge.
[(CO2,CO5)(Analyse/IOCQ)]
(4 + 4) + 4 = 12

Group – D

6. (a) We are given a 0-1 knapsack problem with 6 items as follows: (1,3,8), (2,4,7), (3,2,5), (4,5,4), (5,3,6), (6,4,9), where each triple (u,v,w) specifies the item number, its weight and its value in left-to-right order. The knapsack has a weight limit of 13. Solve the problem using dynamic programming and obtain the maximum total value that can be achieved. *[(CO3)(Apply/HOCQ)]*

(b) Solve the Fractional Knapsack Problem with the data given in (a). Show the change in the solution if it is specified that item 1 must be included entire (i.e., fully, not partially) in the knapsack. *[(CO3)(Apply/LOCQ)]*
7 + 5 = 12

7. (a) Let A_1, A_2, A_3, and A_4 be four matrices of dimension 10 × 5, 5 × 20, 20 × 10, and 10 × 5, respectively. What will be the minimum number of scalar multiplications required to find the product of $A_1A_2A_3A_4$? How will the matrices be parenthesized? (*Do not forget to show the intermediate steps*).
[(CO2, CO3) (Understand/LOCQ)]

(b) Write the pseduo code of Floyd-Warshall's algorithm for computing all pair shortest path in an weighted undirected graph. State the time complexity with justification. Suppose you have an algorithm A for computing single source shortest path. In which situation (with respect to number of verties and edges of

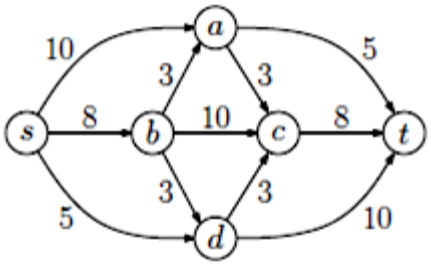
G and edge weights) algorithm A can be used to compute all pair shortest path more efficiently than Floyd-Warshalls' algorithm.

[(CO2,CO4,CO5)(Understand,Analyse/LOCQ)]

6 + (3 + 3) = 12

Group - E

8. (a) Define a flow network. Apply FORD-FULKERSON algorithm on the following flow network to find the maximum flow in the network. s & t denote source & destination and the integer number associated with every edge represents capacity of that edge.



[(CO4)(Remember,Apply/IOCQ)]

(b) In order to prove a problem to be NP-Complete what you need to show?

[(CO2,CO3)(Understand/LOCQ)]

(2 + 7) + 3 = 12

9. (a) Define the VERTEX COVER Problem.
Let G = (V,E) be an undirected graph with n > 1 vertices. Suppose each pair of vertices in V is connected by an edge (i.e., G is a complete graph). What is the size of the smallest vertex cover of G? Explain your answer. *[(CO3)(Apply/HOCQ)]*

(b) Write an approximation algorithm to solve the vertex cover problem. How you measure the quality of the solution produced by your algorithm with respect to the optimal solution? *[(CO6)(Apply/IOCQ]*

4 + 8 = 12

Cognition Level	*LOCQ*	*IOCQ*	*HOCQ*
Percentage distribution	*44*	*41*	*15*

Course Outcome (CO):

After the completion of the course students will be able to

1. Remember time complexities of various existing algorithms in different situations
2. Understand the basic principles of different paradigms of designing algorithms
3. Apply mathematical principles to solve various problems
4. Analyze the complexities of various algorithms
5. Evaluate the performance of various algorithms in best case, worst case and average case
6. Create/ Design a good algorithm for a new problem given to him/ her.

**LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.*