ADVANCED NUMERICAL METHODS
(MATH 2202)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The singular values of the matrix $\begin{pmatrix} 0 & 1 & 1 \\ \sqrt{2} & 2 & 0 \\ 0 & 1 & 1 \end{pmatrix}$ are

(a) $\sqrt{2}, 2\sqrt{2}$ (b) $1, 0$ (c) $\sqrt{2}, 1$ (d) $2, 2\sqrt{2}$.

(ii) The value of $\|A\|$, when $A = \begin{bmatrix} 1 & -1 & 7 \\ -6 & 8 & -3 \\ 2 & 4 & 2 \end{bmatrix}$ is

(a) 9 (b) 15 (c) 17 (d) 8.

(iii) Geometrically Simpson's one third rule for three points of interpolation represents a

(a) parabola (b) straight line (c) circle (d) cubic polynomial.

(iv) By Lagrange's interpolation method, the value of $f(0)$ for the following data

x:	1	3	4
$f(x)$:	4	12	19

is

(a) 7 (b) 3 (c) -3 (d) 2.

(v) In Gauss-Jordan method, the given system of linear equations represented by $AX = B$ is converted to another system $PX = Q$ where P is

(a) diagonal matrix (b) identity matrix
(c) upper triangular matrix (d) lower triangular matrix.

(vi) $\Delta e^x =$

(a) $e^{x+h} - e$ (b) $e^x - e^{x-h}$ (c) $e^x - e^{x+h}$ (d) $e^x(e^h - 1)$

(vii) Which one of the following statements is false?

(a) $\Delta \equiv E - 1$ (b) $\nabla \equiv 1 - E^{-1}$
(c) $\Delta ab^x = ab^x(b+1)$ (d) $\Delta\left(\frac{1}{x}\right) = -\frac{h}{x(x+h)}$

(viii) $[x, x_0, x_1] = ?$

(a) $\frac{[x,x_0]-[x_0,x_1]}{x-x_1}$ (b) $\frac{[x,x_0]-[x_0,x_1]}{x_1-x}$ (c) $\frac{[x_0,x_1]+[x,x_0]}{x+x_1}$ (d) $\frac{[x,x_1]-[x,x_0]}{x_1-x_0}$

(ix) Suppose we need to calculate the eigenvalues of $A = \begin{pmatrix} 1 & -1 & 2 \\ -1 & 2 & 7 \\ 2 & 7 & 5 \end{pmatrix}$, but due to some errors in calculation of the entries of A, we work with $\hat{A} = \begin{pmatrix} 1.01 & -1.05 & 2.1 \\ -1.05 & 1.97 & 7.1 \\ 2.1 & 7.1 & 4.9 \end{pmatrix}$. The maximum difference in the calculated eigenvalues of $\hat{A}$ and the eigenvalues of A is

(a) 0.12435 (b) 0.23664 (c) 0.36432 (d) 0.14871

(x) The reduction ratio in Dichotomous Search algorithm, if we go through r iterations is

(a) $2^{r/2}$ (b) $2^{r/3}$ (c) 2^{r^2} (d) 2^r.

Group - B

2. (a) Solve the given system, using Gauss elimination method with partial pivoting

$$2x + 10y + z = 13$$
$$10x + y + z = 12$$
$$x + y + 3z = 5$$

[(MATH2202.1, MATH2202.4, MATH2202.6)(Evaluate/HOCQ)]

(b) What is condition number of a matrix? Find the condition number of the system of linear equations $AX = B$, where $A = \begin{bmatrix} 2 & 1 \\ 2.01 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 2.05 \end{bmatrix}$, using the infinite norm. Is the above system ill-conditioned? Justify your answer.

[(MATH2202.1, MATH2202.4, MATH2202.6)(Analyze/IOCQ)]

6 + 6 = 12

3. (a) Solve the following system of equations by Cholesky's Decomposition method.

$$x + 2y + 3z = 20$$
$$2x + 8y + 22z = 15$$
$$3x + 22y + 82z = 5$$

[(MATH2202.1, MATH2202.4, MATH2202.6)(Apply/IOCQ)]

(b) Find the inverse of the following matrix, using Gauss-Jordan method.

$$\begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 2 \end{bmatrix}$$

[(MATH2202.1, MATH2202.4, MATH2202.6)(Remember/LOCQ)]

6 + 6 = 12

Group - C

4. (a) Find the QR factorization of $A = \begin{pmatrix} -1 & -1 & 1 \\ 1 & 3 & 3 \\ -1 & -1 & 5 \\ 1 & 3 & 7 \end{pmatrix}$.

[(MATH2202.3, MATH2202.4, MATH2202.6)(Understand/LOCQ)]

(b) Compute three iterations of power method to approximate the dominant eigenvector of $A = \begin{pmatrix} -7 & 2 \\ 8 & -1 \end{pmatrix}$, taking the initial approximation $X_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. Also find the corresponding eigenvalue using Rayleigh quotient.

[(MATH2202.3, MATH2202.4, MATH2202.6)(Evaluate/HOCQ)]

5 + 7 = 12

5. Find the singular value decomposition of the matrix $A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$.

[(MATH2202.3, MATH2202.4, MATH2202.6)(Apply/IOCQ)]

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Group - D

6. (a) Obtain the cubic spline approximation for the function in [0, 1] defined by the data:

x:	0	1	2	3
$f(x)$:	1	2	33	244

with $M_0 = 0, \quad M_3 = 0$, where $M_{x_i} = f''(x_i)$.

[(MATH2202.2, MATH2202.6)(Evaluate/HOCQ)]

(b) Find $f(x)$ as a polynimial in x with the following table using Newton's divided difference formula:

x	-1	0	1	2	3	4
$f(x)$	-16	-7	-4	-1	8	29

[(MATH2202.2, MATH2202.6)(Apply/IOCQ)]

6 + 6 = 12

7. (a) A solid of revolution is formed by rotating about the $x - axis$, the lines $x = 0$ and $x = 1$ and a curve through the points with the following co-ordinates:

x:	0.00	0.25	0.50	0.75	1.00
y :	1.000	0.9896	0.9589	0.9089	0.8415

Estimate the volume formed using Simpson's one third rule.

[(MATH2202.2, MATH2202.6)(Apply/IOCQ)]

(b) Evaluate the integral $\int_0^{\frac{\pi}{2}} \sqrt{1 - 0.162\, sin^2 x}\, dx$ by Weddle's rule.

[(MATH2202.2, MATH2202.6)(Evaluate/HOCQ)]

6 + 6 = 12

Group - E

8. Derive the value of Golden ratio and use the Golden Section Search technique to minimize the function $f(x) = 4x^3 + x^2 - 7x + 14$ in the interval [0,1] taking tolerance to be less than 0.15. *[(MATH2202.5, MATH2202.6)(Apply/IOCQ)]*

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9. Use Dichotomous search algorithm to minimize the function $f(x) = x^2(x - 3.5)$, over [0,1] using tolerance 0.15. Consider $\in = 0.001$. *[(MATH2202.5, MATH2202.6)(Apply/IOCQ)]*

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Cognition Level	*LOCQ*	*IOCQ*	*HOCQ*
Percentage distribution	*23.96*	*56.25*	*19.79*

Course Outcome (CO):

After the completion of the course students will be able to

MATH2202.1 Analyze certain algorithms, numerical techniques and iterative methods that are used for solving system of linear equations.

MATH2202.2 Implement appropriate numerical methods for solving advanced engineering problems dealing with interpolation, integration and differentiation.

MATH2202.3 Apply the knowledge of matrices for calculating eigenvalues and eigenvectors and their stability for reducing problems involving Science and Engineering

MATH2202.4 Develop an understanding to reduce a matrix to its constituent parts in order to make certain subsequent calculations simpler.

MATH2202.5 Apply various optimization methods for solving realistic engineering problems.

MATH2202.6 Compare the accuracy and efficiency of the above mentioned methods.

**LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.*