MATHEMATICS II
(MATH 1201)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The rate of convergence of bisection method is
(a) linear (b) quadratic (c) biquadratic (d) cubic.

(ii) If A and B are independent events such that $P(B) = \frac{2}{7}$ and $P(A + \bar{B}) = 0.8$, where $\bar{B}$ is the complement of the event B, then $P(A)$ is
(a) 0.1 (b) 0.2 (c) 0.3 (d) 0.4.

(iii) If all the element of the second row of an incidence matrix $I(G)$ are zero then the second vertex $v_2$is
(a) pendant (b) isolated
(c) odd (d) of degree 3.

(iv) Laplace transform of $f(t) = te^{2t}$ is
(a) $\frac{1}{s-2}$ (b) $2(s-2)^2$
(c) $\frac{1}{(s-2)^2}$ (d) $\frac{1}{s+2}$.

(v) A complete graph must be a
(a) cycle (b) regular graph
(c) non-simple graph (d) bipartite graph.

(vi) If a binary tree has 20 pendant vertices then the number of internal vertices of the tree is
(a) 20 (b) 21 (c) 23 (d) 19.

(vii) Gauss-elimination method fails when any one of the pivotal elements is
(a) 0 (b) 1 (c) 2 (d) -1.

(viii) The value of $\int_0^\infty e^{-x} x^{3/2}\, dx$ is
(a) $\frac{3}{4}\sqrt{\pi}$ (b) $\frac{3}{5}\sqrt{\pi}$ (c) $\frac{5}{4}\sqrt{\pi}$ (d) $\frac{1}{4}\sqrt{\pi}$.

(ix) A random variable X has the following probability density function
$$f(x) = \begin{cases} k, & -2 < x < 1 \\ 0, & elsewhere \end{cases},$$
then the value of the constant k is

(a) $\frac{1}{8}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{12}$.

(x) If $L\{f(t)\} = \frac{1}{s+1}$, then $L\{f(3t)\}$ is

(a) $\frac{3}{s+3}$ (b) $\frac{1}{s+3}$ (c) $\frac{1}{3s+1}$ (d) $\frac{3}{3s+1}$.

Group - B

2. (a) It is known that glucose level in the blood of diabetic persons follows a normal distribution model with mean 106 mg / 100 ml and standard deviation 8 mg / 100 ml.
   (i) Calculate the probability of a random diabetic person having a glucose level less than 120 mg / 100 ml.
   (ii) What percentage of persons have glucose level between 90 mg/100 ml and 120 mg/100 ml? *[(MATH1201.1, MATH1201.2) (Evaluate /HOCQ)]*

   (b) The distance (in kilometers) travelled by a cyclist in a day is a continuous random variable X whose cumulative distribution function (c.d.f.) is given by:
$$F(x) = \begin{cases} 0 & , \quad x < 0 \\ \dfrac{x^3}{216} & , \quad 0 \le x < 6 \\ 1 & , \quad x \ge 6 \end{cases}$$
   What is the average distance travelled by the cyclist? Further find the standard deviation of the distance travelled by the cyclist from the average distance.
   [(MATH1201.1, MATH1201.2)(Analyse/IOCQ)]

6 + 6 = 12

3. (a) A random variable X follows a binomial distribution with mean 4 and standard deviation $\sqrt{2}$. Find the probability of assuming the non-zero values of the variable. *[(MATH1201.1, MATH1201.2) (Understand/LOCQ)]*

   (b) It is estimated that 50% of emails are spam emails. Some software has been applied to filter these spam emails before they reach your inbox. Its accuracy for detecting a spam mail is 99% of the spam emails (a spam email detected as spam), and the probability for a false positive (a non-spam email detected as spam) is 5%. Now if an email is detected as spam, then what is the probability that it is in fact a non- spam email? *[(MATH1201.1, MATH1201.2)(Understand/LOCQ)]*

6 + 6 = 12

Group - C

4. (a) Solve the following system of linear equations by Gauss-Seidel method, correct to 3 decimal places.
$$x + 4y - z = 6$$
$$x - y + 5z = 7$$
$$6x + y + z = 20$$
   [(MATH1201.3) (Apply/IOCQ)]

(b) Using Newton-Raphson method evaluate $\sqrt[5]{3}$, correct upto four decimal places.
[(MATH1201.3) (Apply/IOCQ)]

6 + 6 = 12

5. (a) Solve the following system of equations by Gauss-elimination method:

$$2x + 8y + 2z = 14$$
$$x + 6y - z = 13$$
$$2x - y + 2z = 5$$

[(MATH1201.3) (Understand/LOCQ)]

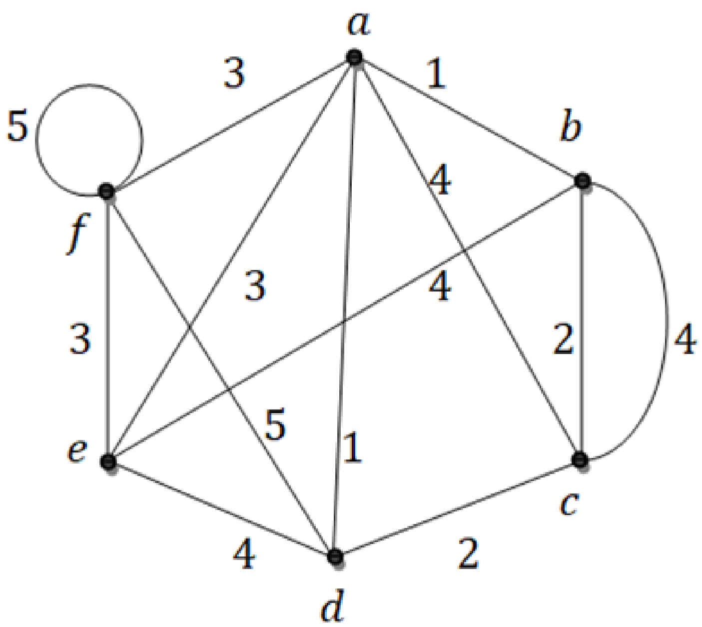
(b) Using Runge-Kutta method of fourth order, find $y(1.4)$ for $\frac{dy}{dx} = 3x + y^2$, $y(1) = 1$ by taking $h = 0.2$.
[(MATH1201.3)(Apply/IOCQ)]

6 + 6 = 12

Group - D

6. (a) A non-digraph G has 6 vertices each of degree 3 and remaining vertices have degree less than 3. Find the minimum number of vertices G may have.
[(MATH1201.4)(Evaluate/HOCQ)]

(b) Find the minimal spanning tree and its weight by Kruskal's algorithm for the following graph:



[(MATH1201.4)(Apply/IOCQ)]

6 + 6 = 12

7. (a) If a simple regular graph has n vertices and 24 edges, find all possible values of n.
[(MATH1201.4)(Analyze/IOCQ)]

(b) Construct the graph whose incidence matrix is given as

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array}.$$

[(MATH1201.4) (Understand/LOCQ)]

(c) Let T be a tree with 32 edges. Removal of one edge from T, two disjoint trees T_1 and T_2 are obtained. If number of vertices of T_1 is twice the number of edges in T_2, find the number of edges in T_1 and T_2. *[(MATH1201.4) (Understand/LOCQ)]*

6 + 3 + 3 = 12

Group - E

8. (a) Using Laplace transformation solve the following initial value problem:

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} = \sin 2t\,, \qquad y(0) = 0,\ y'(0) = 1\,.$$

[(MATH1201.5, MATH1201.6) (Understand /LOCQ)]

(b) Evaluate: $\int_0^1 \frac{dx}{(1-x^6)^{\frac{1}{6}}}$. *[(MATH1201.5, MATH1201.6)(Evaluate/HOCQ)]*

6 + 6 = 12

9. (a) Express the function $f(t) = \begin{cases} t-1, & 1 < t < 2 \\ 3-t, & 2 < t < 3 \end{cases}$
in terms of unit step function and hence find $L\{f(t)\}$.
[(MATH1201.5, MATH1201.6) (Apply/IOCQ)]

(b) Evaluate $L^{-1}\left\{\frac{4s+5}{(s-4)^2(s+3)}\right\}$. *[(MATH1201.5, MATH1201.6)] (Apply/IOCQ)]*

(c) Find $L\{e^t \sin t \cos t\}$. *[(MATH1201.5, MATH1201.6) (Understand /LOCQ)]*

(2 + 4) + 4 + 2 = 12

Cognition Level	*LOCQ*	*IOCQ*	*HOCQ*
Percentage distribution	*33.33*	*47.92*	*18.75*

Course Outcome (CO):

After the completion of the course students will be able to

MATH1201. 1. Demonstrate the knowledge of probabilistic approaches to solve wide range of engineering problem.

MATH1201. 2. Recognize probability distribution for discrete and continuous variables to quantify physical and engineering phenomenon.

MATH1201. 3. Develop numerical techniques to obtain approximate solutions to mathematical problems where analytical solutions are not possible to evaluate.

MATH1201. 4. Analyze certain physical problems that can be transformed in terms of graphs and trees and solving problems involving searching, sorting and such other algorithms.

MATH1201. 5. Apply techniques of Laplace Transform and its inverse in various advanced engineering problems.

MATH1201. 6. Interpret differential equations and reduce them to mere algebraic equations using Laplace Transform to solve easily.

**LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.*