FINITE ELEMENT METHOD
(MECH 3231)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) From the following, which type of element is not two dimensional?
(a) Tetrahedron (b) Quadrilateral
(c) Parallelogram (d) Rectangle.

(ii) Shape function is just a ___________
(a) Displacement function (b) Equation
(c) Interpolation function (d) Matrix function.

(iii) $U_e = 1/2\int \sigma^T \varepsilon A \, dx$ is a ____________
(a) Potential equation (b) Element strain energy
(c) Load (d) Element equation.

(iv) The elemental stiffness matrix of a BAR element having cross-sectional area 'A', elemental length 'L' and modulus of elasticity 'E' is
(a) $\frac{L}{AE}\begin{bmatrix}-1 & 1\\ -1 & 1\end{bmatrix}$ (b) $\frac{AE}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$
(c) $\frac{AL}{E}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$ (d) $\frac{AE}{L}\begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}$

(v) Elemental degree of freedom of a plane FRAME element is
(a) 3 (b) 4
(c) 5 (d) 6.

(vi) The coefficient in stress-strain relation for a linear, elastic, isotropic material under plane strain condition is given by
(a) $\frac{E}{(1+\nu)}\begin{bmatrix}1-\nu & 0 & 0\\ \nu & 1-\nu & 0\\ 0 & 0 & \frac{1-2\nu}{2}\end{bmatrix}$ (b) $\frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu & 0\\ \nu & 1-\nu & 0\\ 0 & 0 & \frac{1-2\nu}{2}\end{bmatrix}$
(c) $\frac{E}{(1-2\nu)}\begin{bmatrix}1-\nu & 0 & 0\\ \nu & 1-\nu & 0\\ 0 & 0 & \frac{1-2\nu}{2}\end{bmatrix}$ (d) $\frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}\nu & 0 & 0\\ \nu & \nu & 0\\ 0 & 0 & \frac{1-2\nu}{2}\end{bmatrix}$

(vii) Plane trusses are also known as ________
(a) one–dimensional trusses (b) two-dimensional trusses
(c) three-dimensional trusses (d) poly dimensional trusses.

(viii) Which of the following method is used for numerical integration?
(a) Newton Raphson method (b) Navier Stokes method
(c) Gauss Legendry method (d) Rayleigh Ritz method.

(ix) For two-point numerical integration by Gauss Quadrature formula, the locations of sampling point (x_i) are
(a) $\left(+\frac{1}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right)$ (b) $\left(+\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{5}}\right)$
(c) $\left(+\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$ (d) $\left(+\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$.

(x) Which one of the following software a numerical software based on Finite Element Method?
(a) 3D MAX (b) TALLY
(c) MAYA (d) MSC Software.

Group- B

2. For the following differential equation and stated boundary conditions, obtain a one-term solution using Galerkin's method of weighted residuals using the specified trial function.

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 3, \qquad 0 \leq x \leq 1$$

The given boundary conditions are

$$y(0) = 0$$
$$y(1) = 0$$

The trial function is: $N_1(x) = x^2(1-x)$. *[(CO1)(Assess/IOCQ)]*

12

3. For the assembly of springs as shown Fig.1 below, determine global stiffness matrix of the whole spring assembly system using Rayleigh-Ritz Method and also find out deflection of the cart at junction 3.

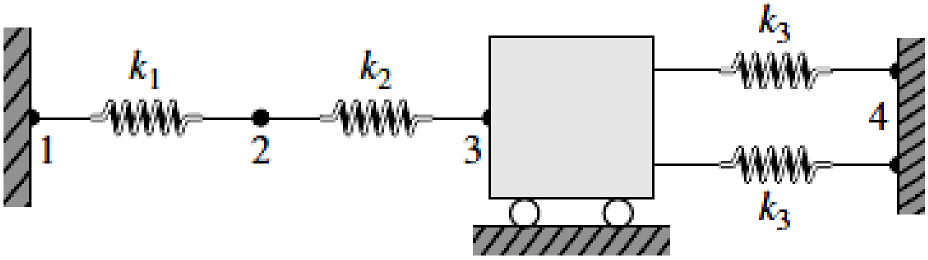

Fig. 1

[(CO2)(Assess/IOCQ)]

(8 + 4) = 12

Group - C

4. Fig.2 shows a two-member plane truss supported by a linearly elastic spring. The truss members are of a solid circular cross section having d = 25 mm and E = 100 GPa. The linear spring has stiffness constant 100 N/mm. Now determine global stiffness matrix and hence write down final FEA formulation to determine (i) displacement of the unconstrained node, (ii) reaction forces at constrained nodes and (iii) stresses in the members.

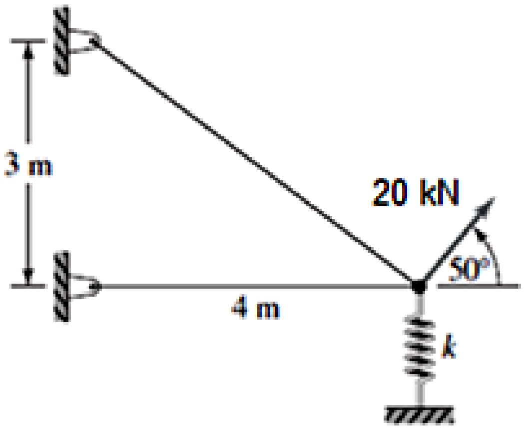

Fig.2

[(CO3)(Analyse/HOCQ)]
6 + (2 + 2 + 2) = 12

5. Consider the beam shown in Fig.3 below. The beam is made of mild steel of grade 40C8 having Modulus of Elasticity 200 GPa and Yield Strength 380 MPa. Under the given loading and boundary conditions determine the global stiffness matrix and hence construct the final FEA formulation to determine deflection of the beam at mid of Uniformly Distributed Load (UDL). Also write down the necessary FEA formulation to determine stresses in the beam at the above-mentioned point. Consider Poisson's ration as 0.31. Take 2 equal elements. You need not solve the equations.

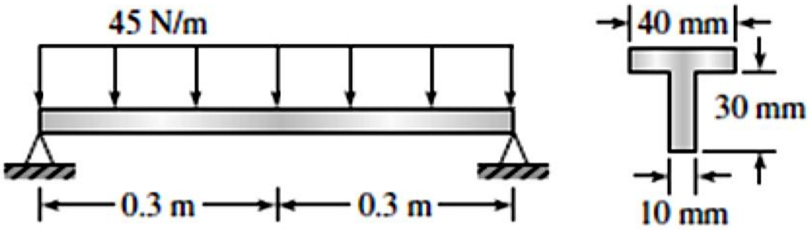

Fig.3

[(CO3)(Estimate/HOCQ)]
6 + 3 + 3 = 12

Group - D

6. Schematically represent a Quadrilateral element showing its nodal degree freedom in user coordinate system. Also derive the expressions of its shape functions in user coordinate system.
[(CO4)(Assess/IOCQ)]
(4 + 8) = 12

7. What is normalized coordinate system? Compare it with User Coordinate system. Write down expressions of shape functions of a 2-node BAR element and 3-node BAR element in Normalized coordinate system.
[(CO5)(Understand/IOCQ)]
(3 + 4 + 5) = 12

Group - E

8. (a) Fig.4 shows a quadrilateral element in global coordinates. Show that the mapping correctly describes the line connecting nodes 2 and 3 and determine the (x, y) coordinates corresponding to $(\xi, \eta,)$ = (0.7, 0.4).

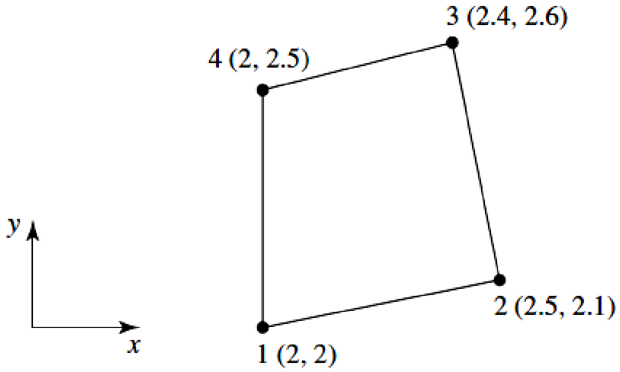

Fig. 4 *[(CO5)(Estimate/IOCQ)]*

(b) Evaluate the following integral using two-point Gaussian quadrature. Compare each result with the corresponding analytical solution

$$\int_0^5 \int_0^6 (2x + 5y)dxdy.$$

[(CO5)(Estimate/IOCQ)]

(3 + 3) + (3 + 3) = 12

9. Describe in detail about Pre-Processing and Post-Processingsteps used in a FEA software. Your answer should be accompanied with various schematic representations.

[(CO6)(Remember/IOCQ)]

(6 + 6) = 12

Cognition Level	*LOCQ*	*IOCQ*	*HOCQ*
Percentage distribution	*0*	*75*	*25*

Course Outcome (CO):

On completion of this course students will be able to

1. Choose suitable material of a product to be designed as per the application and strength requirement.
2. Relate relevant 'Mode of Failure' and 'Theory of Failure' when solving a problem regarding design of machine components under different types of loadings and boundary conditions.
3. Identify proper stress intensity factors for objects with dimensional discontinuity subjected to different loadings and boundary conditions.
4. Analyse life of a machine component with or without dimensional discontinuity subjected to various dynamic loadings constrained with different boundary conditions.
5. Evaluate detailed specifications for fasteners like screw, nut-n-bolt, for welding and power screw by analysing the machine component subjected to various loading and boundary conditions.
6. Design a solid and hollow shaft, coil and leaf spring, shaft couplings and various belts for a belt drive for given power rating, loadings and boundary conditions.

**LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.*