COMPUTATIONAL FLUID DYNAMICS
(MECH 3221)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) In the context of a second-order PDE for a 2D problem, a characteristic is defined as
(a) A line across which the first order derivatives are discontinuous
(b) A surface across which the first order derivatives are discontinuous
(c) A line across which the second order derivatives are discontinuous
(d) A surface across which the second order derivatives are discontinuous.

(ii) Consider the following statements
I. Pressure force is a type of surface force
II. Pressure force is a type of body force
III. Gravity is a type of body force
IV. Coriolis force is a type of body force
Which of the following statement(s) is/are correct?
(a) Only (I) and (III) (b) Only (II) and (III)
(c) Only (II) and (IV) (d) (I), (III) and (IV)

(iii) Consider the following steps
I. Division of the domain into a number of sub-domains, with each sub-domain represented by a finite number of grid points
II. Integration of the governing differential equation over each sub-domain
III. Conversion of the governing differential equation into algebraic expressions using Taylor's series expansion
IV. Profile assumption for the dependent variable for evaluating the integral, in order to express the results as algebraic expressions at the grid points.
The finite volume method involves
(a) Step (I) only (b) Steps (I) and (II) only
(c) Steps (I), (III) and (IV) only (d) Steps (I), (II) and (III) only

(iv) Choose the correct statement on the nature of partial differential equations
(a) If the equation has no real characteristics, the equation is elliptic.
(b) If the equation has no real characteristics, the equation is hyperbolic.
(c) If the equation has no real characteristics, the equation is parabolic.
(d) If the equation has one real characteristics, the equation is elliptic.

(v) Which of the following statements is incorrect in case of finite volume discretization?
(a) If the source term is linearized as $S = S_C + S_P T_P$ then S_P may be of any sign.
(b) If the source term is linearized as $S = S_C + S_P T_P$ then S_P should be negative.
(c) If the source term is linearized as $S = S_C + S_P T_P$ then S_C should be negative.
(d) If the source term is linearized as $S = S_C + S_P T_P$ then S_C should be positive.

(vi) Consider the following statements pertaining to the solution for a system of line algebraic homogeneous equations:
I. Solution is trivial when the determinant of the coefficient matrix is not equal to zero
II. Solution is trivial when the determinant of the coefficient matrix is equal to zero
III. Infinite number of solution exist when the determinant of the coefficient matrix is equal to zero
IV. Infinite number of solution exist when the determinant of the coefficient matrix is not equal to zero
Which of the following statements are correct?
(a) (I) only (b) (II) only (c) (I) and (III) (d) (II) and (IV).

(vii) Which of these does not characterize a turbulent flow?
(a) Time-independence (b) Rapid mixing
(c) Three-dimensional fluctuations (d) Instability.

(viii) Consistency physically means
(a) Nullification of truncation error as grid size and time step size tend to zero
(b) Nullification of round-off error as grid size and time step size tend to zero
(c) Nullification of discretization error as grid size and time step size tend to zero
(d) Nullification of discretization error.

(ix) A staggered grid system is used mainly to
(a) Check the propagation of round-off error
(b) Enable treatment of flow domain of irregular shapes
(c) Check the propagation of truncation error
(d) Eliminate possibilities of highly irregular checker-board pressure field.

(x) Tetrahedral computational cells have
(a) 5 faces (b) 2 faces (c) 3 faces (d) 4 faces.

Group - B

2. (a) State the different forces acting on fluid particles in fluid flow system, with two examples of each. [(CO1)(Understand/LOCQ)]

(b) Calculate the substantial derivative $\frac{D\vec{V}}{Dt}$ of the 3D velocity field represented by $\vec{V} = (-2x)\hat{i} + (-3y)\hat{j} + (4z)\hat{k}$ at a point P (1, -3, 4). State the physical significance of the obtained result. [(CO1)(Apply/IOCQ)]

(2 + 2) + (5 + 3) = 12

3. (a) Derive the relation $\rho\frac{D\phi}{Dt} = \frac{\partial(\rho\phi)}{\partial t} + \nabla\cdot\left(\rho\phi\vec{V}\right)$, where symbols have their usual meanings. [(CO2)(Remember/LOCQ)]

(b) In the general momentum transport equation, what is the physical significance of the term $\frac{\partial\tau_{ji}}{\partial x_j}$ in indicial notation? Write down the long-hand expression of this term in Cartesian coordinates for the z-component of the momentum equation. [(CO2)(Understand/LOCQ)]

6 + 6 = 12

Group - C

4. (a) Consider 1D, steady-state, source-free heat conduction in an insulated metallic rod of 0.8 m length, whose ends are maintained at constant temperature of 150°C and 450°C respectively. The thermal conductivity and cross sectional area of the rod are $k = 1000\ \frac{W}{mK}$ and A = 0.01 m^2 respectively. Find out the set of algebraic equations for the temperature distribution along the rod using finite volume method, by dividing the rod into four equal control volumes. [(CO3)(Analyze/IOCQ)]

(b) Write a short note on boundedness of a discretization scheme for handling convection-diffusion coupled problems. [(CO4)(Analyze/IOCQ)]

8 + 4 = 12

5. (a) Write a short note on conservativeness of a discretization scheme for handling convection-diffusion coupled problems. [(CO4)(Understand/LOCQ)]

(b) Compare the central difference and the upwind approximations for convection-diffusion problems. [(CO4)(Analyze/IOCQ)]

4 + 8 = 12

Group - D

6. Solve the following matrix equation using the TDMA:

$$\begin{bmatrix} 375 & -125 & 0 & 0 & 0 \\ -125 & 250 & -125 & 0 & 0 \\ 0 & -125 & 250 & -125 & 0 \\ 0 & 0 & -125 & 250 & -125 \\ 0 & 0 & 0 & -125 & 375 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix} = \begin{bmatrix} 29000 \\ 4000 \\ 4000 \\ 4000 \\ 54000 \end{bmatrix}$$

[(CO5)(Apply/IOCQ)]

12

7. (a) The 'SIMPLER' algorithm an improvement over the SIMPLE' algorithm: Justify. [(CO5)(Evaluate/HOCQ)]

   (b) Describe the 'SIMPLER' algorithm in flowchart form. [(CO5)(Design/HOCQ)]

4 + 8 = 12

Group - E

8. (a) A square cavity has uniformly higher temperature on the left wall and uniformly lower temperature on the right wall. The top and the bottom walls of the cavity are adiabatic. Write out the detailed steps to solve this fluid flow problem using CFD software. [(CO6)(Apply/IOCQ)]

   (b) For the above problem, write the steps needed to obtain the dimensionless temperature contour. [(CO6)(Apply/IOCQ)]

8 + 4 = 12

9. Write short notes on (i) Pre-processor (ii) User-defined Function (iii) Meshing. [(CO6)(Understand/LOCQ)]

(4 + 4 + 4) = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	33.33	54.17	12.50

Course Outcome (CO):

After the completion of the course students will be able to

1. Describe the fundamental conservation laws of fluid mechanics
2. Express the transport equations in general form
3. Construct the methodologies for converting Partial Differential Equations (PDE) to discretized algebraic forms using Finite Volume Method (FVM)
4. Analyze various CFD solution algorithms for steady and unsteady flows
5. Compare the results obtained from direct analytical solution and FVM using Tri-Diagonal Matrix Algorithm (TDMA)
6. Formulate CFD problems using CFD software and examine the validity of such schemes.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question