MATHEMATICS II
(MATH 1201)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) If a connected simple graph has 8 vertices and e edges then,
(a) $7 \le e \le 26$ (b) $6 \le e \le 28$
(c) $7 \le e \le \infty$ (d) $7 \le e \le 28$

(ii) Determine the value of k so that the lines $\frac{x-1}{2}=\frac{y-4}{1}=\frac{z-5}{2}$ and $\frac{x-2}{-1}=\frac{y-8}{k}=\frac{z-11}{4}$ may intersect
(a) 2 (b) 4
(c) 3 (d) 5

(iii) $B(1,1)=?$, where $B(m,n)$ represents the beta function.
(a) 1 (b) 0
(c) -1 (d) 2

(iv) Choose the correct statement
(a) every walk is a path
(b) every circuit is a path
(c) every loop is a circuit
(d) the original and the terminal vertices of a walk are always distinct.

(v) $L\{3t\}=?$
(a) $\frac{3}{s}$ (b) $\frac{3}{s^2}$
(c) 3 (d) $3s$

(vi) The direction cosines of the line joining the points (2,3,4) and (1,5,6) are
(a) $\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}$ (b) $-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$
(c) $-\frac{1}{3}, -\frac{1}{3}, -\frac{2}{3}$ (d) 1, 1, 1

(vii) The distance of the point (2,3,4) from the planes $3x + 4y + 5z + 7 = 0$ is

(a) 9 (b) $\frac{9}{\sqrt{3}}$

(c) $\frac{9}{\sqrt{2}}$ (d) $\frac{1}{9}$

(viii) A complete graph may not be

(a) regular (b) connected

(c) simple (d) circuit

(ix) The general solution of $\frac{d^2y}{dx^2} - y = 0$ is

(a) $y = ae^{x} + be^{-x}$ (b) $y = (a + bx)e^{x}$

(c) $y = ax^2 + bx + c$ (d) $y = a\cos x + b\sin x$

Where a, b, c are arbitrary constants

(x) $\frac{1}{D^2} sin2x =$

(a) $\frac{1}{4} sin2x$ (b) $sin2x$

(c) $-\frac{1}{2} sin2x$ (d) $-\frac{1}{4}\sin 2x$

Group- B

2. (a) Solve $(1 + xy)ydx + (1 - xy)xdy = 0$.

(b) Solve the differential equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{3x}$ using D-operator method.

6 + 6 = 12

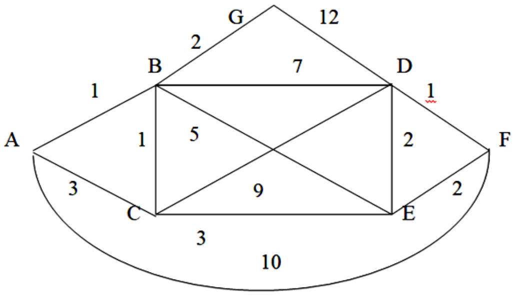
3. (a) Solve: $y = 2px + y^2p^3$, where $p \equiv \frac{dy}{dx}$.

(b) Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2} + 4y = \sin 2x$.

6 + 6 = 12

Group - C

4. (a) Using Dijkstra's algorithm find the shortest path and the length of the shortest path between vertices A and D in the following graph.



(b) If G be a bipartite graph with 22 vertices with partite sets U and V where U contains 12 vertices. Suppose every vertex of U has degree 3 while every vertex of V has degree either 2 or 4. How many vertices of G have degree 2?

7 + 5 = 12

5. (a) Draw the graph whose adjacency matrix is

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(b) Show that the number of odd degree vertices in any graph is always even.

6 + 6 = 12

Group - D

6. (a) Evaluate $\int_0^\infty \frac{sint}{t}\, dt$ by finding the Laplace transform of $F(t) = \frac{sint}{t}$

(b) Show that $\int_0^{\frac{\pi}{2}} \sqrt{tanx}\, dx = \frac{\pi}{\sqrt{2}}$.

6 + 6 = 12

7. (a) Evaluate $L^{-1}\left\{\frac{1}{s(s+1)}\right\}$

(b) Using Laplace transform solve the following initial value problem:

$$\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 9y = e^{3t}, y(0) = 2, y^{/}(0) = 6.$$

6 + 6 = 12

Group - E

8. (a) Find the equation of the image of the line $\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{2}$ in the plane $2x - y + z + 3 = 0$.

(b) Prove that the acute angle between the lines whose d.cs. are given by the relations $l + m + n = 0 \; and \; l^2 + m^2 - n^2 = 0$ is $\frac{\pi}{3}$.

6 + 6 = 12

9. (a) Find whether the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $4x - 3y + 1 = 0 = 5x - 3z + 2$ are coplanar and if so then find the equation to the plane in which they lie.

(b) Perpendiculars PL, PM, PN are drawn from the point $P(a, b, c)$ to the coordinate planes.
Show that the equation of the plane LMN is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 2$.

6 + 6 = 12