

**NUMERICAL & STATISTICAL METHODS**  
**(MATH 2002)**

**Time Allotted : 3 hrs**

**Full Marks : 70**

*Figures out of the right margin indicate full marks.*

*Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.*

*Candidates are required to give answer in their own words as far as practicable.*

**Group - A**  
**(Multiple Choice Type Questions)**

1. Choose the correct alternative for the following: **10 × 1 = 10**
- (i) The order of convergence of bisection method is  
(a) linear                      (b) quadratic                      (c) cubic                      (d) 1.5.
- (ii) The expected value of the product of points on two dice, both tossed simultaneously is  
(a)  $\frac{7}{2}$                       (b)  $\frac{2}{7}$                       (c)  $\frac{4}{49}$                       (d)  $\frac{49}{4}$ .
- (iii) The mean and variance of a Binomial distribution  $B(n, p)$  are 10 and  $\frac{10}{3}$  respectively. Then the value of  $n$  and  $p$  are  
(a)  $(15, \frac{2}{3})$                       (b)  $(15, \frac{1}{3})$                       (c)  $(10, \frac{2}{3})$                       (d)  $(10, \frac{1}{3})$
- (iv) If  $X$  and  $Y$  are independent of each other, then  $r_{xy}$  is  
(a)  $-1$                       (b)  $0$                       (c)  $1$                       (d)  $0.5$ .
- (v) A system of equations  $AX = b$  where  $A = (a_{ij})_{n \times n}$  is said to be diagonally dominant if  
(a)  $|a_{ii}| \geq \sum_{\substack{j=1 \\ i \neq j}}^n |a_{ij}|$  for all  $i$                       (b)  $|a_{ii}| < \sum_{\substack{j=1 \\ i \neq j}}^n |a_{ij}|$  for all  $i$   
(c)  $|a_{ii}| > \sum_{j=1}^n |a_{ij}|$  for all  $i$                       (d)  $|a_{ii}| < \sum_{j=1}^n |a_{ij}|$  for all  $i$ .
- (vi) If two events  $A$  and  $B$  are mutually exclusive, then  $P(A \cap B)$  is  
(a)  $-0.01$                       (b)  $0$                       (c)  $1$                       (d)  $0.5$ .
- (vii) Newton backward interpolation formula is used for  
(a) equal intervals                      (b) unequal intervals  
(c) both equal & unequal intervals                      (d) even no. of intervals.
- (viii)  $\Delta^n x^n = ?$   
(a)  $n!$                       (b)  $(n - 1)!$                       (c)  $n^2$                       (d)  $0$ .

- (ix) The mode and median of the observation 4, 6, 6, 8, 3, 8, 8 & 4 are  
 (a) 8 and 6                      (b) 8.5 and 6.5                      (c) 5 and 7                      (d) 4 and 3.
- (x) A matrix A can be factorized into lower and upper triangular matrix if all the principal minors of A are  
 (a) singular                      (b) non-singular                      (c) zero                      (d) have determinant value  $\pm 1$ .

**Group - B**

2. (a) Solve the following system of equations by Gauss-elimination method:  

$$\begin{aligned} x + 3y + 2z &= 5 \\ 2x - y + z &= -1 \\ x + 2y + 3z &= -2 \end{aligned}$$
*[[Understand/LOCQ]]*
- (b) Compute, by the Newton-Raphson method, the root of the equation  $x^3 - x - 1 = 0$  that lies between  $x = 1$  and  $x = 2$ , correct to three places of decimal. *[[Apply/IOCQ]]*  
**6 + 6 = 12**
3. (a) Find the smallest positive root of the equation  $xe^x - 1 = 0$ , correct to 3 significant figures using bisection method. *[[Apply/IOCQ]]*
- (b) Solve the following system of linear equations, correct to 2 decimal places, by Gauss Siedel method.  

$$\begin{aligned} x + 10y + z &= 20.88 \\ 10x + y - z &= 11.19 \\ -x + y + 10z &= 35.61 \end{aligned}$$
*[[Apply/IOCQ]]*  
**6 + 6 = 12**

**Group - C**

4. (a) Calculate  $f(5)$  by using Lagrange's interpolation formula, given the following data.
- |        |    |    |    |    |
|--------|----|----|----|----|
| $x$    | 3  | 7  | 9  | 12 |
| $f(x)$ | 11 | 17 | 24 | 30 |
- [[Understand/IOCQ]]*
- (b) Solve  $\frac{dy}{dx} = x^2 + y^2, y(0) = 0$  using Euler's method and find  $y(0.1)$  &  $y(0.2)$  using  $h = 0.05$ . *[[Remember/LOCQ]]*  
**6 + 6 = 12**
5. (a) Compute the value of  $\pi$  from the formula  $\frac{\pi}{4} = \int_0^1 \frac{1}{1+x^2} dx$  using Trapezoidal rule taking the 10 sub-intervals. *[[Apply/IOCQ]]*
- (b) Compute  $f(0.29)$  by Newton's backward difference interpolation formula from the following table:
- |        |        |        |        |        |        |        |
|--------|--------|--------|--------|--------|--------|--------|
| $x$    | 0.20   | 0.22   | 0.24   | 0.26   | 0.28   | 0.30   |
| $f(x)$ | 1.6596 | 1.6698 | 1.6804 | 1.6912 | 1.7024 | 1.7139 |
- [[Evaluate/HOCQ]]*
- 
- 6 + 6 = 12**

**Group - D**

6. (a) Two newspapers  $X$  and  $Y$  are published in a certain city. It is estimated from a survey that 16% read  $X$ , 14% read  $Y$  and 5% read both the newspapers. Find the probabilities that a randomly selected person
- (i) does not read any newspaper
  - (ii) read only  $Y$ . [[Understand/IOCQ]]
- (b) If  $A$  and  $B$  are two independent events, then prove that  $A^c$  and  $B^c$  are also independent. [[Remember/LOCQ]]
- 6 + 6 = 12**
7. (a) There are two identical urns containing 4 white and 3 red balls; 3 white and 7 red balls. An urn is chosen at random and a ball is drawn from it. Find the probability that the ball is white. What is the probability that it is from the first urn if the ball drawn is white? [[Apply/IOCQ]]
- (b) A bag contains 5 white and 4 black balls. If 3 balls are drawn at random, what are the probabilities of the following?
- (i) two of them are white
  - (ii) at most one of them is white
  - (iii) at least two are white. [[Understand/IOCQ]]
- 6 + 6 = 12**

**Group - E**

8. (a) Assuming that the height distribution of a group is normal, find the mean and standard deviation if 84% of the men have heights less than 65.2 inches and 68% have heights lying between 62.8 and 65.2 inches.
- [Given  $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0.9} e^{-\frac{t^2}{2}} dt = 0.84$  and  $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-0.9} e^{-\frac{t^2}{2}} dt = 0.16$ ] [[Apply/IOCQ]]
- (b) For two variables  $x$  and  $y$  the equations of two regression lines are  $x + 2y + 5 = 0$  and  $2x + 3y + 8 = 0$ . Identify which one is the regression line of  $y$  on  $x$ . Find the means of  $x$  &  $y$ . Find the correlation coefficient between  $x$  and  $y$ . [[Analyze/IOCQ]]
- 6 + 6 = 12**
9. (a) Show that the function  $f(x)$  given by
- $$f(x) = \begin{cases} x, & 0 \leq x < 1 \\ k - x, & 1 \leq x < 2 \\ 0, & \text{otherwise} \end{cases}$$
- is a probability density function, for a suitable value of the constant  $k$ . Construct the distribution function of a random variable  $X$  and compute the probability that the random variable  $X$  lies between  $\frac{1}{2}$  and  $\frac{3}{2}$ . [[Evaluate/HOCQ]]
- (b) A random variable  $X$  follows binomial distribution with mean 4 and standard deviation  $\sqrt{2}$ . Find the probability of assuming the non-zero value of the variable. [[Understand/LOCQ]]
- 6 + 6 = 12**

<i>Cognition Level</i>	<i>LOCQ</i>	<i>IOCQ</i>	<i>HOCQ</i>
<i>Percentage distribution</i>	43.75	43.75	12.5

*\*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question*