MATHEMATICAL METHODS
(MATH 2001)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The function $f(z) = \frac{1}{2z}$ is not analytic at

(a) $z = \frac{1}{2}$ (b) $z = 0$ (c) $z = 1$ (d) $z = \pi$.

(ii) Let C be the circle $|z| = 1$, then $\int_C \frac{\sin 4z}{z-\frac{\pi}{4}}\, dz$ is

(a) 3 (b) π (c) 0 (d) $\frac{i\pi}{4}$.

(iii) If $f(x) = \begin{cases} -x + i(y - x) & if\ \ 0 \le x < \pi \\ 25 & if\ \pi \le x \le 2\pi \end{cases}$ and $f(x) = f(x + 2\pi)$, then the sum of the Fourier series at $x = \pi$ is

(a) 0 (b) 25 (c) $\frac{25}{2}$ (d) 1.

(iv) The value of $J_0(0)$ is [where $J_n(x)$ is the Bessel's function]

(a) 1 (b) 0 (c) -1 (d) ± 1.

(v) The point $z = 0$ of the function $\frac{\sin z}{z^3}$ is

(a) simple pole (b) removable singularity

(c) pole of order 2 (d) pole of order 3.

(vi) The function $f(z) = |\bar{z}|^2$ is

(a) nowhere continuous (b) continuous everywhere

(c) not analytic (d) continuous everywhere except at $z = 0$.

(vii) $\frac{d}{dx}\{x^4 J_4(x)\} =$

(a) $x^4 J_5(x)$ (b) $x^4 J_3(x)$ (c) $x^3 J_4(x)$ (d) $-x^3 J_4(x)$.

(viii) The value of $\int_C \frac{dz}{z+5}$, where C is the circle $|z| = \pi$, is

(a) 0 (b) π (c) 2πi (d) 10πi.

(ix) The singular point(s) of the ordinary differential equation $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$ is/are

(a) 0 (b) only 1 (c) $1, -1$ (d) only -1.

(x) The partial differential equation $p \tan y + q \tan x = \sec^2 z$ is of order

(a) 0 (b) 1 (c) 2 (d) 3.

Group - B

2. (a) Show that the function defined by $f(z) = \sqrt{|xy|}$ is not analytic at the origin although Cauchy-Riemann equations are satisfied at the origin.

[(MATH2001.1, MATH2001.2)(Analyze/IOCQ)]

(b) Use Cauchy's integral formula to calculate $\int_C \frac{2z-1}{z(z+6)}\,dz$, where $C: |z| = 4$.

[(MATH2001.1, MATH2001.2)(Apply/IOCQ)]

6 + 6 = 12

3. (a) Show that the function $f(z) = -x + i(y-x)$ is everywhere continuous but not analytic. *[(MATH2001.1, MATH2001.2)(Apply/IOCQ)]*

(b) Find the value of p so that the function $(2x - x^2 + py^2)$ is harmonic.

[(MATH2001.1, MATH2001.2)(Understand/LOCQ)]

(c) Evaluate $\int_C \frac{3z^2+z-1}{(z^2-1)(z-3)}\,dz$, where $C: |z| = 2$, using Cauchy's residue theorem.

[(MATH2001.1, MATH2001.2)(Evaluate/HOCQ)]

4 + 2 + 6 = 12

Group - C

4. (a) Find the Fourier series of the function *f(x)* with period 2π defined by

$$f(x) = \begin{cases} -1, & -\pi < x \le 0 \\ 1, & 0 \le x < \pi \end{cases}$$

Also obtain the sum of the series at $x = \pm\pi$ and show that, $1 - \frac{1}{3} + \frac{1}{5} - \ldots$ to $\infty = \frac{\pi}{4}$ *[(MATH2001.1, MATH2001.3, MATH2001.4) (Evaluate/HOCQ)]*

(b) Find the Fourier transform of $xe^{\frac{-25x^2}{2}}$.

[(MATH2001.1, MATH2001.3, MATH2001.4)(Analyze/IOCQ)]

7 + 5 = 12

5. (a) Find the half-range Fourier cosine series of the function $f(x) = x - 1$ in $0 < x < 1$.

Hence use Parseval's identity to prove that $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots = \frac{\pi^4}{96}$.

[(MATH2001.1, MATH2001.3, MATH2001.4) (Apply/IOCQ)]

(b) Find the inverse Fourier transform of $\frac{1}{s^2+6s+18}$.

[(MATH2001.1, MATH2001.3, MATH2001.4)(Remember/LOCQ)]

7 + 5 = 12

Group - D

6. (a) Find the series solution of $\frac{d^2y}{dx^2} + (x-1)\frac{dy}{dx} + y = 0$ about the point $x = 0$.
[(MATH2001.5)(Evaluate/HOCQ)]

(b) Express $J_4(x)$ in terms of $J_0(x)$ and $J_1(x)$. *[(MATH2001.5)(Evaluate/HOCQ)]*

8 + 4 = 12

7. (a) Define the Bessel's function of 1st kind of order n and hence prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$. *[(MATH2001.5)(Apply/IOCQ)]*

(b) Prove that $nP_n(x) = xP_n'(x) - P_{n-1}'(x)$, where $P_n(x)$ is the Legendre polynomials of degree n. *[(MATH2001.5)(Remember/LOCQ)]*

6 + 6 = 12

Group - E

8. (a) Form the partial differential equation from the relation $\varphi\left(\frac{z}{x^3}, \frac{y}{x}\right) = 0$ by eliminating the arbitrary function φ. *[(MATH2001.1, MATH2001.6)(Apply/IOCQ)]*

(b) Apply Lagrange's method to solve the following linear partial differential equation $y^2p - xyq = x(z + 2y), p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$.
[(MATH2001.1, MATH2001.6)(Apply/IOCQ)]

6 + 6 = 12

9. (a) Solve the following partial differential equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \cos x \cos 2y$.
[(MATH2001.1, MATH2001.6)(Analyze/IOCQ)]

(b) Solve $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial z}{\partial y} = 2\frac{\partial z}{\partial x}$ by the method of separation of variables.
[(MATH2001.1, MATH2001.6)(Create/HOCQ)]

6 + 6 = 12

Cognition Level	*LOCQ*	*IOCQ*	*HOCQ*
Percentage distribution	*13.54*	*54.17*	*32.29*

Course Outcome (CO):

After the completion of the course students will be able to

MATH2001.1 Construct appropriate mathematical models of physical systems.

MATH2001.2 Recognize the concepts of complex integration, Poles and Residuals in the stability analysis of engineering problems.

MATH2001.3 Generate the complex exponential Fourier series of a function and make out how the complex Fourier coefficients are related to the Fourier cosine and sine coefficients.

MATH2001.4 Interpret the nature of a physical phenomena when the domain is shifted by Fourier Transform e.g. continuous time signals and systems.
MATH2001.5 Develop computational understanding of second order differential equations with analytic coefficients along with Bessel and Legendre differential equations with their corresponding recurrence relations.
MATH2001.6 Master how partial differentials equations can serve as models for physical processes such as vibrations, heat transfer etc.

**LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.*