COMPUTATIONAL MATHEMATICS
(MATH 3221)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The greatest common divisor of the Fibonacci numbers F_{15} and F_{16} is
(a) 3 (b) 1 (c) 2 (d) 4.

(ii) The fourth harmonic number H_4 is
(a) $\frac{11}{6}$ (b) $\frac{3}{2}$ (c) $\frac{25}{12}$ (d) $\frac{137}{60}$.

(iii) The Eulerian number $E(3,1)$ is
(a) 3 (b) 2 (c) 1 (d) 4.

(iv) The remainder in the division of $1! + 2! + 3! + 4! + 5! + 6! + \cdots + 100!$ by 24 is
(a) 10 (b) 12 (c) 7 (d) 9

(v) The coefficient of x^{12} in $(1 + x)^{20}$ is
(a) C(20,12) (b) $C(32,12)$ (c) $C(12,8)$ (d) C(20,8).

(vi) The Stirling number $S_2(4, 2)$ is
(a) 1 (b) 7 (c) 6 (d) 8.

(vii) $C(100, 30) + C(100, 29) =$
(a) $C(101, 29)$ (b) $C(101, 31)$ (c) $C(101, 30)$ (d) $C(100, 31)$.

(viii) The minimum number of moves that will transfer 20 disks from one peg to another in the Tower of Hanoi problem is
(a) 524287 (b) 2097151 (c) 4194303 (d) 1048575.

(ix) Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x. If $c > 0$ and $\lfloor \sqrt[10]{c} \rfloor = 1$, then
(a) $1500 < c < 2000$ (b) $c < 1$ (c) $1 < c < 1500$ (d) $c > 2000$.

(x) The generating function of the sequence $1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, \dots$ is

(a) $\frac{1}{(1-x)^2}$ (b) $\frac{1}{1-x^2}$ (c) $\frac{1}{1+x^2}$ (d) $\frac{1}{(1+x)^2}$.

Group- B

2. (a) Prove that $\frac{x^{\underline{m}}}{(x-n)^{\underline{m}}} = \frac{x^{\underline{n}}}{(x-m)^{\underline{n}}}$, unless one of the denominators is zero.

[(MATH3221.1, MATH3221.5, MATH3221.6)(Understand/LOCQ)]

(b) Use the perturbation technique to prove that $S_n := \sum_{0 \le k \le n} k2^k = (n -$ *12n+1+2.* *[(MATH3221.1, MATH3221.5, MATH3221.6)(Analyse/IOCQ)]*

5 + 7 = 12

3. (a) Compute $\Delta^6(x^8)$ and $\nabla^2(x^{\bar{8}})$, where $\Delta f(x) = f(x+1) - f(x)$ and $\nabla f(x) = f(x) - f(x-1)$.

[(MATH3221.1, MATH3221.5, MATH3221.6)(Analyze/IOCQ)]

(b) Compute $\sum_{0 \le n \le 5} \frac{1}{3n+1}$ and $\sum_{0 \le n^2 \le 5} \frac{1}{3n^2+1}$. Show your calculations in detail.

[(MATH3221.1, MATH3221.5, MATH3221.6)(Remember/LOCQ)]

6 + 6 = 12

Group - C

4. (a) Prove that $\sum_{0 \le k \le n} C(m+k, k) = C(m+n+1, n)$, where m and n are positive integers. *[(MATH3221.2, MATH3221.5, MATH3221.6)(Analyse/IOCQ)]*

(b) State the recurrence relation for $S_1(n, k)$, the Stirling numbers of the first kind. Use it to compute $S_1(4,1)$, $S_1(4,2)$, $S_1(4,3)$, $S_1(5,1)$, $S_1(5,2)$, $S_1(5,3)$.

[(MATH3221.2, MATH3221.5, MATH3221.6)(Remember/LOCQ)]

6 + 6 = 12

5. (a) Let F_n denote the n-th Fibonacci number. Prove that (i) $F_{n+6} = 8F_{n+1} + 5F_n$. (ii) $F_{n-4} = -3F_{n+1} + 5F_n$. *[(MATH3221.2, MATH3221.5, MATH3221.6)(Remember/LOCQ)]*

(b) Prove the recurrence relation $E(n,k) = (k+1)E(n-1,k) + (n-k)E(n-1,k-1)$ for the Eulerian numbers $E(n,k)$, where n is a positive integer.

[(MATH3221.2, MATH3221.5, MATH3221.6)(Apply/IOCQ)]

(3 + 3) + 6 = 12

Group – D

6. (a) Find the remainder in the division of $3^{721} + 36! + 35!$ by 37.
Show your calculations in detail and state every theorem that you use.

[(MATH3221.3, MATH3221.5, MATH3221.6)(Evaluate/HOCQ)]

(b) (i) Prove that $\lfloor x+n \rfloor = \lfloor x \rfloor + n$, where n is an integer.

(ii) Show that $\lfloor x+y \rfloor$ is equal to either $\lfloor x \rfloor + \lfloor y \rfloor$ or $\lfloor x \rfloor + \lfloor y \rfloor + 1$, where x and y are positive real numbers.

[(MATH3221.3, MATH3221.5, MATH3221.6)(Evaluate/HOCQ)]

6 + (3 + 3) = 12

7. (a) Let $\varphi(n)$ denote the Euler phi function. Compute $\varphi(36), \varphi(25), \varphi(29)$. Show your work and state any result that you use.

[(MATH3221.3, MATH3221.5, MATH3221.6)(Evaluate/HOCQ)]

(b) Let $a \equiv b \pmod{m}, c \equiv d \pmod{m}$. Prove that $ac \equiv bd \pmod{m}$ and $a^8 \equiv b^8 \pmod{m}$.

[(MATH3221.3, MATH3221.5, MATH3221.6)(Apply/IOCQ)]

6 + 6 = 12

Group – E

8. (a) Use the method of generating function to solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + 4^n;\ n \geq 2$, given that $a_0 = 2, a_1 = 8$.

[(MATH3221.4, MATH3221.5, MATH3221.6)(Evaluate/HOCQ)]

(b) Solve the following recurrence relation: $a_n = 4a_{n-1} - 4a_{n-2} + (n+1)2^n$.

[(MATH3221.4, MATH3221.5, MATH3221.6)(Apply/IOCQ)]

6 + 6 = 12

9. (a) Find a generating function for
$a_r =$ the number of ways the sum r can be obtained when:
(i) 2 distinguishable dice are tossed;
(ii) 2 distinguishable dice are tossed and the first shows an even number and the second shows an odd number;
(iii) 10 distinguishable dice are tossed and 6 specified dice show an even number and the remaining 4 show an odd number.

[(MATH3221.4, MATH3221.5, MATH3221.6)(Create/HOCQ)]

(b) Find the coefficient of X^{20} in $(X^3 + X^4 + X^5 + \cdots)^5$.

[(MATH3221.4, MATH3221.5, MATH3221.6)(Analyse/IOCQ)]

6 + 6 = 12

Cognition Level	*LOCQ*	*IOCQ*	*HOCQ*
Percentage distribution	*23.96*	*44.79*	*31.25*

Course Outcome (CO):

After the completion of the course students will be able to
MATH3221.1. Identify patterns in data in the form of recurrences and use them to evaluate finite and infinite sums.
MATH3221.2. Explain combinatorial phenomena by using binomial coefficients and special numbers.
MATH3221.3. Solve computational problems by applying number theoretic concepts such as primality, congruences, residues etc.
MATH3221.4. Use generating functions to study diverse computational phenomena.
MATH3221.5. Combine the concepts of recurrences, sums, combinatorics, arithmetic etc. in order to comprehend computational concepts.
MATH3221.6. Interpret mathematically the algorithmic features of computational situations.

**LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.*