COMPUTATIONAL METHODS OF ENGINEERING
(MECH 4142)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The convergence criteria for Gauss Seidel method requires that diagonal element of coefficient matrix must be
(a) larger than summation of other elements of the row
(b) smaller than summation of other elements of the row
(c) exactly equal to summation of other elements of the row
(d) very smaller than the summation of other elements of the row.

(ii) Compared to analytical method, numerical method provides
(a) faster but less accurate result
(b) faster and accurate result
(c) difficult and less accurate result
(d) difficult but accurate result.

(iii) The number of significant digit in 2.4305 is
(a) 4 (b) 5 (c) 3 (d) 6.

(iv) In two successive iterations, if the first iteration provides a value of 3.4256 and the second one provides 3.4268, the percentage relative error of approximation is
(a) 0.235 (b) 0.568 (c) 0.035 (d) 0.147.

(v) Lagrange interpolating polynomial is the rearrangement of
(a) Newton's interpolating polynomials
(b) Newton's forward interpolating method
(c) Newton's backward interpolating method
(d) None of the above.

(vi) In LU decomposition method of solution the coefficient matrix is decomposed into
(a) lower and upper triangular matrix both
(b) lower triangular matrix only
(c) upper triangular matrix only
(d) none but constant coefficient matrix is decomposed into both.

(vii) Through certain data points, if a cubic polynomial is to fit, the numbers of regression equation will be
(a) 5 (b) 4 (c) 2 (d) 3.

(viii) The order and degree of the ordinary differential equation $\frac{d^3y}{dx^3} + \sqrt{\frac{dy}{dx}} + y = 0$ are respectively
(a) 3; 1 (b) 3; 2 (c) 2; 3 (d) 1; 3.

(ix) The order and degree of the ordinary differential equation $\left(\frac{dy}{dx}\right)^3 = \sqrt{\left(\frac{d^4y}{dx^4}\right)^2 + 1}$ are respectively
(a) 6; 2 (b) 4; 3 (c) 2; 4 (d) 4; 2.

(x) The $2-\mathrm{D}$ Laplace equation $u_{xx} + u_{yy} = 0$ is classified as
(a) elliptic (b) hyperbolic (c) linear (d) parabolic.

Group – B

2. (a) The amount of a uniformly distributed radioactive contaminant contained in a closed reactor is measured by its concentration **c** (becquerel/liter or Bq/L). The decay rate is shown in the following equation where **t** is time in day (d).

$$\frac{dc}{dt} = -kt$$

Use Euler's Numerical method to solve the equation from t = 0d to t=1d with step size 0.2d. The value of k is 0.165 d^{-1}. The Initial value of c is 85 Bq/L.

(b) Explain true error and relative percentage true error with an example.

8 + 4 = 12

3. (a) An electronic company produces transistors, resistors and computer chips. Each transistor, resistor and computer chip required copper, zinc and glass with the quantity written below.

Components	Copper	Zinc	Glass
Transistor	4	1	2
Resistors	3	3	1
Computer chips	2	1	3

In a week, total 940 unit of copper, 530 units of zinc and 600 units of glass were used to produce the components. Using LU decomposition method find out the number of transistors, resistors and computer chips made at that week.

(b) A set of linear equations are written below.

4x+y-z = - 2
5x+y+2z = 4
6x+y+z = 6

Use Gauss elimination process to solve them.

7 + 5 = 12

Group – C

4. (a) The surface area (A) of human beings is related to weight (W) and height (H) as shown below.

H (cm)	182	180	179	187	189	194
W (kg)	74	88	94	78	84	98
A (m^2)	1.92	2.11	2.15	2.02	2.09	2.31

Using Multiple linear regressions, develop an equation and predict the surface area of a man having height 184 cm and weight 78 kg.

(b) The following data defines the sea level concentration (C) of dissolved oxygen for fresh water as a function of temperature (T).

T (°C)	0	8	16	24	32	40
C (mg/L)	14.62	11.84	9.87	8.42	7.31	6.41

Estimate C (29°C) using Newton's interpolating polynomial.

7 + 5 = 12

5. (a) In an experiment, the percentage of elongation of a material is noted with respect to temperature (T), as given below.

T (°C)	200	250	300	375	425	475	600
% elongation	7.5	8.6	8.7	10	11.3	12.7	15.3

Use Newton's polynomial interpolation to predict the percentage of elongation at 400 °C.

(b) Fit a straight line by linear regression method for the following data.

X	6	7	11	15	17	23
y	29	21	29	14	21	7

7 + 5 = 12

Group – D

6. (a) Using Simpson's 1/3rd rule, evaluate the following integral for 6 intervals.

$$\int_{-3}^{9} (4x-3)^4 dx$$

(b) Derive the two point Gauss quadrature formula to solve a definite integral.

5 + 7 = 12

7. (a) Compute the value of $(1.1)^{-2}$ from the Taylor's series expansion of the function $f(x) = \frac{1}{x^2}$ about $x = 1$ and truncated up to four terms.

(b) Determine the displacement $u(x,t)$ of a string if $u_{tt} = u_{xx}, u(x,t=0) = \frac{x(10-x)}{100}, u(x=0,t) = 0, u(x=10,t) = 0, u_t(x,t=0) = 0$ for $x = 0(1)10,\ t = 0(1)5$.

3 + 9 = 12

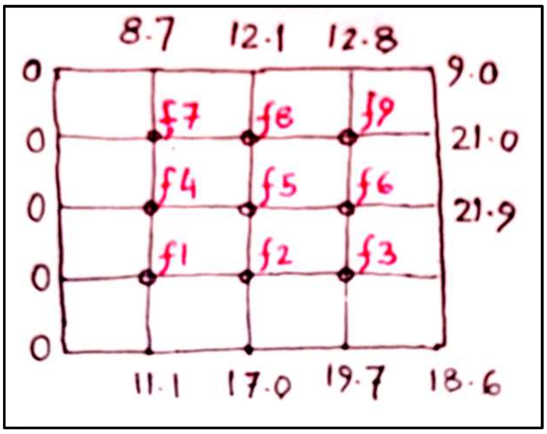
Group – E

8. (a) From experiment we get the following values of the (unknown) function $\boldsymbol{y} = \boldsymbol{\varphi}(\boldsymbol{x})$: $y_0(x_0 = 1) = 3, y_1(x_1 = 2) = -5, y_2(x_2 = -4) = 4$. Compute the Lagrange polynomial $P(x)$ of degree two that represents $\varphi(x)$ approximately.

(b) Given the initial boundary-value problem $\frac{\partial f}{\partial t} = 2\frac{\partial^2 f}{\partial x^2}$; $0 \leq x \leq 6, t > 0$ with the two boundary conditions $f(x = 0, t) = 10$; $f(x = 6, t) = 18$ and the initial condition $f(x, t = 0) = x^2/2$. Consider the spatial increment $h = 1$ and the time increment $k = 1/8$.
Compute the field values for $0 \leq t \leq 5/8$. Estimate the field values as $t \to \infty$.

3 + 9 = 12

9. (a) Compute a solution to the Laplace equation $\nabla^2 f \equiv \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$ at all mesh (grid) points of the following square in which the boundary values are indicated, using the five-point formula.



(b) Consider two approximations of the first-order derivative $f'(x)$ of the function $f(x)$:

(i) $f'(x) \approx \frac{-f(x+2h)+6f(x+h)-3f(x)-f(x-h)}{6h}$

(ii) $f'(x) \approx \frac{-f(x+2h)+8f(x+h)-8f(x-h)+f(x-2h)}{12}$

For $f(x) = \frac{1}{x}$, $h = 0.1$ compute the values of $f'(x = 1)$ by the above two approximations.

9 + 3 = 12

Department & Section	Submission link:
ME	https://classroom.google.com/c/MjQwNjQ0NTcxOTEw/a/Mjc0MDU3MDkxMjcz/details