ADVANCED DISCRETE MATHEMATICS AND STATISTICAL METHODS (MATH 5101)

Time Allotted : 3 hrs   **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and
<u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i)   A simple connected planar graph with the same number of vertices and edges determines
       (a) 1 region   (b) 2 regions   (c) 3 regions   (d) no regions.

(ii)   If $\mathrm{Var}(X) = 1$, then $\mathrm{Var}(-2X + 5)$ is
       (a) -2 (b) 3   (c) -4 (d) 4 .

(iii)   Ramesh has 6 friends. In how many ways can he invite one or more of them at a dinner?
       (a) 61   (b) 62   (c) 63   (d) 64.

(iv)   The probability of an impossible event is
       (a) 0   (b) 1   (c) ∞ (d) 1/2.

(v)   The number of different permutations of the word BANANA is
       (a) 720   (b) 60   (c) 120   (d) 360.

(vi)   Find the mode of the following data:
       $3, 9, 2, 8, 7, 1, 1, 4, 5, 6, 6, 2, 9, 7, 8, 4, 6$.
       (a) 3   (b) 7   (c) 8   (d) 6.

(vii)   If A and B are mutually disjoint, then $P(A \cup B)$ is
       (a) $P(A) + P(B)$ (b) $P(A)P(B)$ (c) $P(A) - P(B)$ (d) $\frac{P(AB)}{P(B)}$.

(viii)   The normal density curve $y = \varphi(x)$ is symmetric about
       (a) $x -$ axis   (b) $y -$ axis
       (c) the line $y = 2$ (d) the line $y = 1$.

(ix)   The chromatic number of a tree is
       (a) 0   (b) 1   (c) 2   (d) 3.

(x)   If the discrete random variable X has the distribution function $F(x)$ and $F(0) = \frac{1}{6}$, $F(1) = \frac{1}{2}$ and $F(3) = \frac{3}{4}$, then $P(0 < X \leq 3)$ is

(a) $\frac{3}{4}$ (b) $\frac{1}{3}$ (c) $\frac{7}{12}$ (d) $\frac{1}{12}$.

Group – B

2. (a)   In a bolt factory, machines A, B, C manufacture respectively 25%, 35% and 40% of the total output of which 5%, 4% and 2% are defective bolts. A bolt is drawn at random from their product and is found to be defective. What is the probability that it was manufactured by machine B?

    (b)   Let a random variable X has the following probability mass function:

x	0	1	2	3	4	5	6	7
$P(X = x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

(i)   Determine the constant k.
(ii) Evaluate $P(X < 6)$, $P(3 < X \leq 6)$.
(iii) Obtain the distribution function $F(x)$.

$$6 + 6 = 12$$

3. (a)   In a certain class 25% of the students failed in mathematics, 15% failed in chemistry, 10% failed in both mathematics and chemistry. A student is selected at random.
    (i)   If he failed in mathematics, what is the probability that he failed in chemistry?
    (ii) If he failed in chemistry, what is the probability that he failed in mathematics?
    (iii) What is the probability that he failed in mathematics or chemistry?

    (b)   The radius of a circle has distribution given by the probability density function
    $$f(x) = \begin{cases} 1, & 1 < x < 2 \\ 0, & elsewhere \end{cases}$$
    Find the mean and variance of the area of the circle.

$$6 + 6 = 12$$

Group – C

4. (a)   A binomial variate $X \sim B(6, p)$ satisfies the relation $9P(X = 4) = P(X = 2)$. Find $P(X = 1)$.

    (b)   100 unbiased coins are tossed. Using normal approximation to binomial distribution, calculate the probability of getting (i) exactly 40 heads (ii) 55 heads or more. Given $\Phi(2.1) = 0.9821$, $\Phi(1.9) = 0.9713$, $\Phi(0.9) = 0.8159$.

$$6 + 6 = 12$$

5. (a)   For two variables x and y, the equations of two regression lines are $x + 2y = 5$ and $2x + 3y = 8$. Identify which one is the regression line of x on y and justify

your answer. Find the correlation coefficient between x and y. Also find the mean of x and mean of y.

(b)   The expenditure of 1000 families is given below:

Expenditure (Rs)	$40 - 59$	$60 - 79$	$80 - 99$	$100 - 119$	$120 - 139$
Frequency	50	?	500	?	50

The median and mean for the distribution are both Rs. 87.50. Calculate the missing frequencies.

6 + 6 = 12

Group – D

6. (a)   How many permutations of the letters A B C D E F G contain
    (i)   the string BCD?
    (ii) the string $CFGA$?
    (iii) the strings BA and GF?
    (iv) the strings ABC and DE?
    (v)   the strings ABC and CDF?
    (vi) the strings CBA and BED?

   (b)   If n pigeonholes are occupied by $(kn + 1)$ pigeons, where k is a positive integer, prove that at least one pigeonhole is occupied by $(k + 1)$ or more pigeons. Hence, find the minimum number m of integers to be selected from $S = \{ 1,2, \dots ,9 \}$ so that the sum of two of the m integers is even.

6 + 6 = 12

7. (a)   Find the number of non-negative integer solutions of the inequality $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 < 10$.

   (b)   Solve the following recurrence relation $a_n = 3a_{n-1} + 1$, $n \geq 1$, given that $a_0 = 1$.

6 + 6 = 12

Group – E

8. (a)   (i)   Can a graph having 150 vertices and 500 edges be planar? Justify your answer.
    (ii) What is the minimum number of edges in a planar graph with 110 faces?
    (iii) A simple connected planar graph with 20 edges determines 8 regions. What is the number of vertices in the graph?

   (b)   Find the chromatic polynomial of the following graphs in Fig.1 and Fig.2:
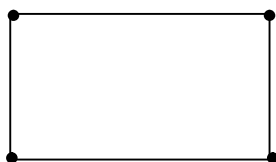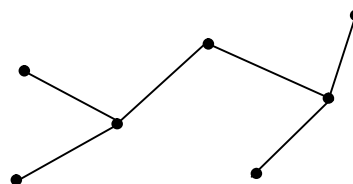   (i)   (ii)

Fig.1   *Fig.2*

(2 + 2 + 2) + 6 = 12

9. (a) (i) If a graph with degree sequence [5,4,4,3,3,3,2,2,2] is planar, find the number of faces in it.
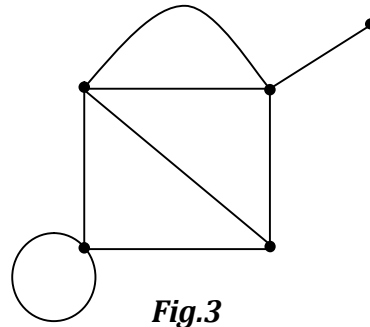   (ii) Draw the dual of the following graph:

Fig.3

(b) Aahana, Bhavya, Charu and Dimple are trying to decide on whom to take to their school prom night. The table below shows which boys they would not mind taking to the prom. Is it possible for all four girls to choose suitable partners? Answer this by setting up a bipartite graph and checking, in detail, to see if Hall's condition is satisfied by the set X consisting of the four girls. If your answer is yes, then find a solution to this problem.

Aahana	Manish, Pratyush
Bhavya	Naman
Charu	Pratyush, Raunak, Sahil
Dimple	Raunak, Sahil

(3 + 3) + 6 = 12

Department & Section	Submission Link
CSE	https://classroom.google.com/c/MjMxMDA1MzkyNDQy/a/MjcxNDM2Mjgz MzU1/details