ADVANCED DATA STRUCTURES
(CSEN 5101)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) A binary search tree T contains the 20 keys {10, 20, 30, . . ., 190, 200}. If it is known that the key at the root of T is 170, then the height of T is at least
(a) 3 (b) 4 (c) 5 (d) 6.

(ii) To sort four integers using insertion sort, the total number of pairwise comparisons in the best case is
(a) 5 (b) 4 (c) 3 (d) 2

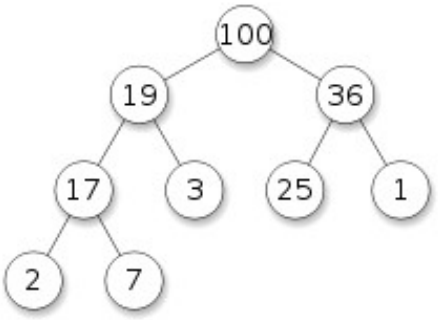
(iii) When red and black colourings are erased from the nodes of a Red-Black (RB) tree and NIL nodes are ignored, the resulting structure is
(a) a binary search tree (b) a height-balanced (AVL) tree
(c) a B-tree (d) no longer a binary tree.

(iv) An arithmetic statement P has single-character operands. The operators are all binary and belong to the set { +,-,*,/,^ }. It is known that P contains just one pair of open-closed parentheses and has no embedded blanks. If P is 21 characters in length, how many operands are there in P?
(a) 11 (b) 10 (c) 9 (d) 8

(v) Time complexity to insert a node at the beginning of a circular singly linked list with only head pointer is
(a) O(1) (b) O(n) (c) O(logn) (d) $O(n^2)$

(vi) In the Knuth-Morris-Pratt (KMP) string matching method, the prefix function array for the pattern 'abcbabbca' is
(a) [0,0,0,0,1,2,2,3,1] (b) [0,0,0,0,1,2,0,0,1]
(c) [0,0,0,1,2,2,3,4,1] (d) [0,0,0,0,1,1,2,3,1]

(vii) Consider the following binary tree



What will be the post order traversal of the binary tree?

(a) 2, 7, 17, 3, 19, 25, 1, 36, 100 (b) 2, 7, 17, 3, 19, 25, 36, 1, 100
(c) 2, 7, 17, 3, 19, 100, 25, 1, 36 (d) 2, 7, 17, 3, 19, 25, 100, 36, 1

(viii) Of the four arrays of 12 positive integers shown below, which one does *not* form a min-heap?

(a) [1,2,4,8,10,12,16,20,25,30,40,50]
(b) [5,10,16,20,15,64,32,40,30,35,25,82]
(c) [9,15,11,20,29,14,13,38,36,34,32,16]
(d) [1,2,6,3,4,8,9,11,12,5,10,7].

(ix) What is the minimum possible height of a B-tree that has 4×10^9 keys if the minimum branching factor t is 1000?

(a) 5 (b) 4 (c) 3 (d) 2

(x) It is given that a binary tree T has 10 internal (i.e., non-leaf) nodes and 6 leaf nodes. Then the total number of edges in T is

(a) 12 (b) 13 (c) 14 (d) 15.

Group – B

2. (a) Consider the following arithmetic statement in infix notation:
p = q r -s + (t – r) ^ (q * s)/t*
The binary arithmetic operators +, -, *, / have their usual meaning; ^ (caret) represents exponentiation and '=' the assignment operator. Single alphabetic characters represent variables. Explain how the above statement can be converted into Polish postfix with the help of a stack.

(b) Define a data structure for a STACK using an array.
Then write an algorithm for the following stack operations:
(i) Push with overflow checking, (ii) Pop with underflow checking.

5 + (1 + 3 + 3) = 12

3. (a) Consider the following sequence of values:
10, 45, 20, 14, 57, 100, 200, 40, 35, 66, 27, 85, 52, 70 and 18.
Suppose we have an array of size 20 and a simple hash function (key % 10) is used to store these values. Two different hash techniques are used:
(i) Separate chaining (ii) Open addressing
State which of the above techniques may suffer more from collisions and why? Then show how you to tackle these collisions.

(b) The following sequence of 12 positive integers is stored in an array S[1..12]:
25, 12, 36, 23, 16, 8, 11, 32, 29, 5, 17, 27. S is to be sorted using heapsort. In the first phase, a max-heap will be created in S. Describe, in a step-by-step manner, how the max-heap gets created.

(1 + 2 + 4) + 5 = 12

Group – C

4. (a) Given the following input sequence of 10 keys:
97 16 25 14 12 4 20 23 1 30
construct a binary search tree T using the binary search and insertion algorithm. Now obtain the post-order traversal sequence of the keys in T.

(b) For the input sequence of keys in (a), construct a Red-Black (RB) tree T_1 by inserting the keys one by one into an initially empty RB tree. What is the black height of T_1? Does there exist an RB tree on the keys in S that has no red nodes?

(4 + 1) + (5 + 1 + 1) = 12

5. (a) A 2-3 tree is a special type of B-tree intended for use in main memory and not intended for storing large files on secondary storage. Let T be a 2-3 tree. Then:
(i) Each node in T, including the root node and the leaf nodes, contains either one key or two keys.
(ii) An internal node with one key has two child nodes, while one with two keys has three child nodes.
(iii) All leaf nodes are at the same level.
(iv) The keys in T are ordered as in a B-tree (i.e., T is a search tree).
Construct a 2-3 tree T of height 2 on the set of 15 keys given by
{5, 12, 15, 22, 37, 40, 43, 45, 51, 61, 63, 65, 70, 80, 84}.

(b) Write a pseudo code for calculating the height of a binary tree, assuming a pointer to the root of the binary tree is supplied.

7 + 5 = 12

Group – D

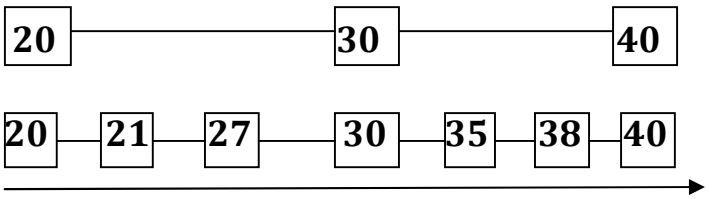
6. (a) Construct a B-tree T with minimum degree t = 3 in a step-by-step manner given the following input sequence of 12 keys: 15, 35, 75, 25, 95, 105, 45, 85, 65, 115, 55, 5.

(b) Why are B-trees preferred for storing data in secondary storage?
When a B-tree is in use by a running program, the root node is usually kept in main memory. What is the advantage of doing so?

8 + (2 + 2) = 12

7. (a) Construct a skip list that has five (or six) levels with the following sorted list of 19 keys:
21 25 33 39 46 51 56 62 68 73 81 89 92 101 110 115 118 125 134

(b) Consider the following 2-level skip list



Explain in Brief how to access element 38 in the above skip list.
What is the time complexity improvement of skip lists from linked lists in insertion and deletion operation? Justify your answer.

6 + (4 + 2) = 12

Group – E

8. (a) Suppose that the procedure 'Compute-Prefix-Function (P)' is available to you. Write the pseudo-code for a KMP matcher (T, P) for text T and pattern P using the above procedure.

(b) Show how you will search the pattern P = '001011000101' in the text T = '011001001011000101100000101' using the naïve (brute-force) pattern-matching algorithm.

6 + 6 = 12

9. (a) In a particular pattern matching problem, we are required to find occurrences of the pattern P **=** "pqrqppqr" in the text T = "qppqrqqprpqrqppqrqppqrqr".
When we try to find all occurrences of pattern P in text T, by the
(i) Simple brute-force method
(ii) KMP algorithm,
What is the number of character comparisons made in each case?

(b) This question relates to Computational Geometry. Let Q = { p_1, p_2, p_3, p_4,p_5 } be a set of five points on the X-Y plane, where p_1 = (2,4), p_2 = (10,8), p_3 = (3,1), p_4 = (6,7,), p_5 = (3,5). Explain how the convex hull CH(Q) of the given set of points can be determined using Graham's scan. (You do not need to perform all the computations.)

(3 + 3) + 6 = 12

Department & Section	Submission Link
CSE	https://classroom.google.com/c/Mjc1NDc2MDk4OTUx/a/MjgzNTgyNDI5OTc5/details