OPTIMIZATION TECHNIQUES
(MATH 6121)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) If the primal contains n variables, then the dual has
(a) n constraints (b) $n-1$ constraints
(c) $n+1$ constraints (d) n^2 constraints.

(ii) Minimize Z =
(a) – Maximize(z) (b) Maximize (z)
(c) Maximize ($-z$) (d) –Minimize ($-z$).

(iii) Matrix minima method is used for solving
(a) assignment problem (b) NLPP
(c) transportation problem (d) game theory.

(iv) The solution (0, ½, 0, 0) of the system of equations
$2x_1 + 6x_2 + 2x_3 + x_4 = 3, 6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$ is:
(a) non-degenerate (b) degenerate
(c) not feasible (d) non-basic.

(v) The optimality condition for a maximization type LPP is
(a) $z_j - c_j \geq 0$ (b) $z_j - c_j \leq 0$
(c) $z_j - c_j < 0$ (d) $z_j - c_j = 1$.

(vi) The value of a for which the following pay-off matrix is strictly determinable

PLAYER B

PLAYER A

a	5	2
-1	a	-8
-2	3	a

(a) $a \leq -1$ (b) $-1 \leq a \leq 2$
(c) for any value of a (d) $a \geq 5$.

(vii) The Hessian matrix of the function $f(x, y, z)$ is

$$H(f(x, y, z)) = \begin{pmatrix} x^2 & 0 & -y^2 \\ 0 & z & y \\ 1 & 0 & z \end{pmatrix}$$

If $(-1, 1, 0)$ is a stationary point of function $f(x, y, z)$, this would be
(a) a local maximum point but not global
(b) a local minimum point but not global
(c) a saddle point
(d) a global minimum point.

(viii) For the standard maximization problem, the Kuhn-Tucker necessary conditions are also sufficient conditions, provided
(a) objective function is convex and constraints are concave
(b) objective function and constraints are concave
(c) objective function and constraints are convex
(d) objective function is concave and constraints are convex.

(ix) Given the optimization problem
O_1 $f(x, y)$ subject to $g(x, y) = b$;
it is known that the bordered Hessian matrix of the Lagrange function $(L(x, y, \lambda))$ when $\lambda = 6$ is given by

$$H^B(L(x, y, \lambda)) = \begin{pmatrix} 0 & 2 & 1 \\ 2 & 4 & 0 \\ 1 & 0 & -6 \end{pmatrix}$$

Then, if $(x, y, \lambda) = (3, 2, 6)$ is a stationary point of the Lagrange function,
(a) the maximum value of the objective function is -6
(b) $(3, 2)$ is a maximum point
(c) $(3, 2)$ is a minimum point
(d) $(3, 2)$ is a saddle point.

(x) Which of the following Hessian matrix belongs to a convex function?

(a) $\begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 2 \\ 1 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$.

Group – B

2. (a) Solve the following linear programming problem by graphical method:

Maximize $z = x_1 + x_2$

Subject to the constraints

$$x_1 - x_2 \geq 0$$
$$2x_1 - x_2 \leq -2$$
$$x_1, x_2 \geq 0$$

(b) Use Simplex method to solve the following linear programming problem:

Maximize $z = 60x_1 + 50x_2$

Subject to

$$x_1 + 2x_2 \leq 40$$
$$3x_1 + 2x_2 \leq 60$$
$$x_1, x_2 \geq 0$$

5 + 7 = 12

3. (a) Use the 'Big-M' method to solve the following linear programming problem:

Minimize $z = 4x_1 + 8x_2 + 3x_3$

Subject to

$$x_1 + x_2 \geq 2$$
$$2x_1 + x_3 \geq 5$$
$$x_1, x_2, x_3 \geq 0$$

(b) Write the dual of the following LPP:

Maximize $z = 2x_1 + 3x_2 - 4x_3$

Subject to

$$3x_1 + x_2 + x_3 \leq 2$$
$$-4x_1 + 3x_3 \geq 4$$
$$x_1 - 5x_2 + x_3 = 5$$
$$x_1, x_2 \geq 0$$

x_3 is unrestricted in sign.

8 + 4 = 12

Group – C

4. (a) Solve the transportation problem by Vogel's approximation method and checking it's optimality, find the optimal solution:

	W_1	W_2	W_3	W_4	Supply
F_1	2	2	2	1	3
F_2	10	8	5	4	7
F_3	7	6	6	8	5
Demand	4	3	4	4	

(b) Solve by North-West corner rule:

	A	B	C	D	Capacity
X	9	8	5	7	12
Y	4	6	8	7	14
Z	5	8	9	5	16
Requirement	8	18	13	3	

8 + 4 = 12

5. (a) Write down the following transportation problem into a linear programming problem form:

	D_1	D_2	a_i
O_1	4	3	7
O_2	5	5	9
b_j	10	6	

(b) Find the assignment of machinists I to IV to jobs A to E in the following matrix that will result in a maximum profit:

	A	**B**	**C**	**D**	**E**
I	6.20	7.80	5.00	10.10	8.20
II	7.10	8.40	6.10	7.30	5.90
III	8.70	9.20	11.10	7.10	8.10
IV	4.80	6.40	8.70	7.70	8.00

5 + 7 = 12

Group – D

6. (a) Use graphical method to solve the following game and find the value of the game:

	Player B			
Player A	1	3	-3	7
	2	5	4	-6

(b) Use dominance to reduce the following pay-off matrix to a 2 × 2 game and hence find the optimal strategies and the value of the game:

		Player B			
		B_1	B_2	B_3	B_4
Player A	A_1	3	2	4	0
	A_2	3	4	2	4
	A_3	4	2	4	0
	A_4	0	4	0	8

6 + 6 = 12

7. (a) Use algebraic method to solve the following game:

	Player B		
Player A	1	-1	-1
	-1	-1	3
	-1	2	-1

(b) In a rectangular game, the pay-off matrix is given by:

	Player B				
Player A	10	5	5	20	4
	11	15	10	17	25
	7	12	8	9	8
	5	13	9	10	5

Find the optimal strategies and the value of the game.

8 + 4 = 12

Group – E

8. (a) Find the nature of the function $f(x) = x^4 + 6x^2 + 12x$.

(b) Solve the following non-linear programming problem using Lagrange multiplier method:

Optimize $z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$

Subject to

$$x_1 + x_2 + x_3 = 15$$
$$2x_1 - x_2 + 2x_3 = 20$$
$$x_1, x_2, x_3 \geq 0$$

3 + 9= 12

9. Use the Kuhn-Tucker conditions to solve the following non-linear programming problem:

Maximize $z = 3x_1 + x_2$

Subject to

$$x_1^2 + x_2^2 \le 5$$
$$x_1 - x_2 \le 1$$
$$x_{1,}\ x_2 \ge 0$$

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Department & Section	Submission Link
CSE	https://classroom.google.com/c/MTE5MDg0NjAyODU3/a/Mjc0MDU1ODM5NTIz/details
ECE	https://classroom.google.com/c/MTE5MDg0NjAyODU3/a/Mjc0MDU1ODM5NTIz/details
VLSI	https://classroom.google.com/c/MTE5MDg0NjAyODU3/a/Mjc0MDU1ODM5NTIz/details