MATHEMATICS - I
(MATH 1101)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and
any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) If A is a square matrix, then $A + A^T$ is
(a) skew symmetric (b) symmetric
(c) orthogonal (d) a unit matrix.

(ii) Let $r(A)$ denote the rank of a matrix A. If all minors of a matrix A of order $r+1$ are zero, then
(a) $r(A) \leq r$ (b) $r(A) = 0$ (c) $r(A) > r$ (d) $r(A) = 1$.

(iii) The series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$
(a) converges to 0 (b) converges to 1
(c) diverges to $-\infty$ (d) diverges to $+\infty$.

(iv) The sequence $1 + \frac{(-1)^n}{n}$, $n = 1, 2, 3, \dots \infty$
(a) converges to 0. (b) diverges to $-\infty$
(c) is monotonically increasing and unbounded below (d) converges to 1.

(v) A vector normal to the plane $x + 2y + 3z - 1 = 0$ is
(a) $2\hat{\imath} + \hat{\jmath} + 3\hat{k}$ (b) $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$
(c) $\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$ (d) $\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$.

(vi) The particular integral of $\frac{d^2y}{dx^2} + y = x^2$ is
(a) $x^2 - 2x + 2$ (b) $x^2 - 2$.
(c) $x^2 + 2$. (d) $x + 2$.

(vii) $\frac{xdy - y}{xy}$ is equal to
(a) $d\left\{\log\left(\frac{y}{x}\right)\right\}$ (b) $d\left\{\log\left(\frac{x}{y}\right)\right\}$
(c) $d\left\{\frac{y}{x}\right\}$ (d) $d\left\{tan^{-1}\left(\frac{y}{x}\right)\right\}$.

(viii) The integrating factor of the equation $\frac{dy}{dx} + 2xy = x^3$ is

(a) x^2 (b) e^{x^2} (c) x^3 (d) e^{2x}.

(ix) $\int_{-a}^{a}\int_{-b}^{b} dxdy$ is equal to

(a) $2a$ (b) $2b$ (c) ab (d) $4ab$.

(x) The order of the homogeneous function $\sin\left(\frac{y}{x}\right) + \tan^{-1}\left(\frac{y}{x}\right)$ is

(a) 1 (b) -1 (c) 0 (d) 2.

Group – B

2. (a) Reduce the matrix A to row-reduced echelon form and hence find its rank, where $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$.

(b) Investigate the solution of the system of linear equations

$$2y + 4z + 5 = 0,$$
$$8x - y + 4z = 12,$$
$$16x - y + 10z = 1.$$

6 + 6 = 12

3. (a) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 9 & 2 & 0 \\ 5 & 0 & 3 \end{bmatrix}$ using Cayley-Hamilton Theorem.

(b) Find the characteristic equation and the eigenvalues of the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

(c) Prove that an orthogonal matrix is non-singular.

5 + 5 + 2 = 12

Group – C

4. (a) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{2^n+1}{3^n+2}$.

(b) Test the convergence of the alternating series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$.

6 + 6 = 12

5. (a) In what direction from the point $(1,1,-1)$ is the directional derivative of $f(x,y,z) = x^2 - 2y^2 + 4z^2$ a maximum? What is the magnitude of this directional derivative?

(b) Find $div\, \vec{F}$ and $curl\, \vec{F}$ where $\vec{F} = grad(x^3 + y^3 + z^3 - 3xyz)$.

5 + 7 = 12

Group – D

6. (a) Show that the following equation is exact and solve it.
$e^x \sin y\, dx + (e^x + 1)\cos y\, dy = 0.$

(b) Solve: $\frac{dy}{dx} + \frac{4x}{x^2+1} y = \frac{1}{(x^2+1)^3}$.

6 + 6 = 12

7. (a) Solve the following differential equation by the D-operator method.
$\frac{d^2y}{dx^2} + 4y = x\, cos x.$

(b) Solve: $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2 \log x.$

6 + 6 = 12

Group – E

8. (a) Show that the double limit does not exist as $(x, y) \to (0,0)$ for the function given by $f(x, y) = \frac{x^3+y^3}{x-y}$

(b) If $u = tan^{-1} \frac{\sqrt{x^3+y^3}}{\sqrt{x}+\sqrt{y}}$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin u \cos u$.

6 + 6 = 12

9. (a) Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2 + y^2)dxdy$ by changing to polar coordinates.

(b) Evaluate $\int_C (2x + y^2)dx + (3y - 4x)dy$ where C denotes the triangle PQR formed by the points $P(0,0)$, $Q(2,0)$, $R(2,1)$ traversed in the clockwise direction.

6 + 6 = 12

Department & Section	Submission Link
AEIE	https://classroom.google.com/c/MjM1NDA1MTY0NzAw/a/Mjc0NDI5ODYyNTQz/details
BT	https://classroom.google.com/c/MjM1NDAzMTkxMjYz/a/Mjc0NDI5ODYyNjE2/details
CHE	https://classroom.google.com/c/MjMwNTc1NzQ2MTM4/a/Mjc0NTMwMjAxODIz/details
CE A	https://classroom.google.com/c/MjMwNTU4ODM4MDA1/a/MjY0ODg4MjI4MDAx/details
CE B	https://classroom.google.com/c/MjMwNTY0MjI2ODcx/a/MjY0ODg4MjI4MTkw/details
CSBS	https://classroom.google.com/c/MjMwNjk3ODM1MDgy/a/Mjc0MDkxMDY4NDU3/details

Department & Section	Submission Link
CSE A	https://classroom.google.com/c/MjMwOTkxNDQxNDgz/a/Mjc0MTQ1NzgxMjI5/details
CSE B	https://classroom.google.com/c/MjMwNTYxOTM3NjM5/a/Mjc0NDk3MzAwNzY4/details
CSE C	https://classroom.google.com/c/MjMwNTYxOTM3NzQ2/a/Mjc0NDk3MzAwNjg2/details
ECE A	https://classroom.google.com/c/MjMwMjAxNjkwMTY1/a/Mjc0MDc5MDMwOTU2/details
ECE B	https://classroom.google.com/c/MjMwNDQ1MDY0MDg1/a/Mjc0MDc4MDk0MDEy/details
ECE C	https://classroom.google.com/c/MjMxMDE1NjgyMjQ1/a/Mjc0NDQzNzI0NzA0/details
EE	https://classroom.google.com/c/MjMwNTc3NjcxNjY1/a/Mjc0NTMxMzAyODA1/details
ME A	https://classroom.google.com/c/MjMxMDA0NDMyODU0/a/Mjc0NDQzNzI1NDc1/details
ME B	https://classroom.google.com/c/MjMwNjg5MjA3NTA3/a/Mjc0MDk3NTA1MDQ4/details
IT	https://classroom.google.com/c/MjA3NTA3NDA5NzM2/a/Mjc0MTUwNzU1NTE4/details

Department & Section	Submission Link
BACKLOG	https://classroom.google.com/c/MjA0OTk4NDc2MDI0/a/MjY1MjAxNzE2Mzgw/details