FINITE ELEMENT METHOD
(MECH 3231)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) Which of the following is an approximate method?
(a) Double integration method (b) Area moment method
(c) Weighted residual method (d) Virtual work method.

(ii) The elemental stiffness matrix of an element can be expressed in generally as
(a) $\int [B]^T\,[B]E\,dv$ (b) $\int [B]\,[B]^T E\,dv$ (c) $\int [B]E\,dv$ (d) $\int [B]\,dv$
All the symbols and notations carry their usual meaning.

(iii) Which of the following is the correct expression of a Shape function of a 2-Node BEAM element among its four shape functions?
(a) $N_1 = 1 - \frac{3x^2}{l^2} + \frac{2x^3}{l^3}$ (b) $N_1 = 1 - \frac{3x^2}{l^2} - \frac{2x^3}{l^3}$
(c) $N_1 = 1 + \frac{3x^2}{l^2} + \frac{2x^3}{l^3}$ (d) $N_1 = 1 - \frac{3x}{l} + \frac{2x^2}{l^2}$

(iv) The elemental stiffness matrix of a spring element having spring constant 'k' is
(a) $k\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$ (b) $k\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ (c) $k\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ (d) $k\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(v) Elemental degree of freedom of a two-dimensional BEAM element is
(a) 2 (b) 3 (c) 4 (d) 5

(vi) The coefficient in stress-strain relation for a linear, elastic, isotropic material under plane stress condition is given by
(a) $\frac{E}{1-\nu^2}\begin{bmatrix} 1 & 0 & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$ (b) $\frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$
(c) $\frac{E}{1-\nu^2}\begin{bmatrix} 1 & 1 & 0 \\ \nu & \nu & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$ (d) $\frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$

(vii) CST element possesses
(a) constant field variable throughout the element
(b) derivative of the field variable is constant throughout the element
(c) variation of the field variable is quadratic throughout the element
(d) variation of the field variable is cubic throughout the element.

(viii) For a Four-Noded Rectangular element if one of the shape functions in user-coordinate (x, y) is expressed as $N_1 = \left(1-\frac{x}{l}\right)\left(1-\frac{y}{w}\right)$ then the corresponding shape function in normalized coordinate (ξ, η) will be?
(a) $\frac{1}{4}(1-\xi)(1+\eta)$ (b) $\frac{1}{4}(1-\xi)(1-\eta)$
(c) $\frac{1}{2}(1-\xi)(1-\eta)$ (d) $\frac{1}{4}(1-\xi)$.

(ix) Isoparametric element is one in which
(a) Both geometry and displacement of element are described by a specific shape function
(b) Both geometry and displacement of element are described by same shape function
(c) Geometry and displacement of element are described by different but related shape function
(d) Shape function for geometry and displacement of element are completely different.

(x) The sequence of the numerical simulation in any FEA software is
(a) Pre-processing → Solution → Post-processing
(b) Post-processing → Solution → Pre-processing
(b) Pre-processing → Post-processing → Solution
(d) Solution → Pre-processing → Post-processing.

Group - B

2. For the following differential equation and stated boundary conditions, obtain a one-term solution using Galerkin's method of weighted residuals using the specified trial function. Compare the approximate solution to the exact solution.

$$\frac{d^2y}{dx^2} + 17y = 3\sin\pi x\,, \qquad 0 \le x \le 1$$

The given boundary conditions are

$$y(0) = 0$$
$$y(1) = 0$$

The trial function is: $N_1(x) = \sin\pi x$. [(CO1)(Assess/IOCQ)]

(8 + 4) = 12

3. For the spring system shown in the accompanying Fig. 1, determine the global stiffness matrix and calculate displacement of each load point. [(CO2)(Estimate/HOCQ)]

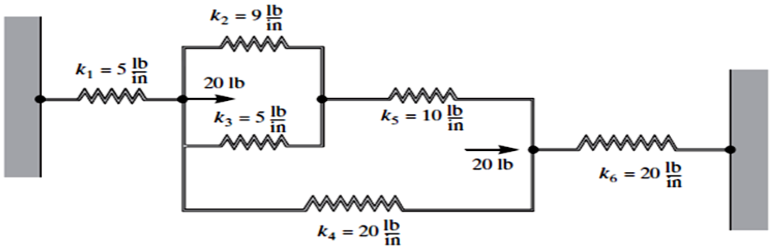

Fig. 1

(7 + 5) = 12

Group - C

4. The plane truss shown in Fig. 2 is composed of members having a square 12 mm × 12 mm cross section. The members are made of MS. Now write down final FEA formulation to determine deflection at joint C and D under the given load. Also mention required FEA formulations to find member reaction forces and member stresses. Modulus of elasticity of truss members' material is given as 210 GPA. [(CO3)(Analyse/HOCQ)]

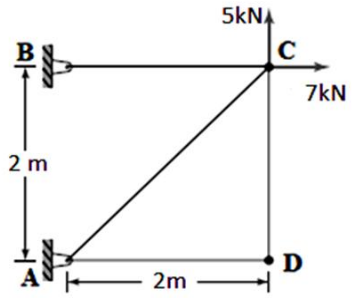

Fig. 2

(4 + 8) = 12

5. (a) The beam element shown in Fig. 3 below is subjected to a linearly varying load of maximum intensity q_0. Using the work-equivalence approach, determine the equivalent nodal forces and moments. [(CO3)(Estimate/IOCQ)]

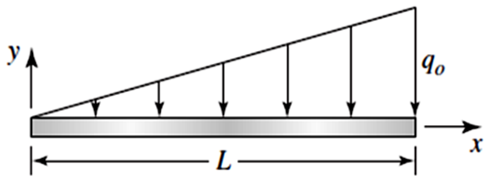

Fig. 3

   (b) Write a short note on CST. [(CO4)(Understand/LOCQ)]

8 + 4 = 12

Group - D

6. Schematically represent a Triangular element showing its nodal degree freedom in user coordinate system. Also derive the expressions of its shape functions in user coordinate system. [(CO4)(Assess/IOCQ)]

(4 + 8) = 12

7. (a) Briefly discuss about plane stress condition with a suitable example. Also write down the stress-strain constitutive relation for a linear, elastic, isotropic material under this plane stress condition. [(CO4)(Remember/LOCQ)]

   (b) What do you understand by 'Normalized Co-ordinate' system? Briefly discuss. Write down the shape functions of a triangular element in normalized coordinate system. [(CO5)(Understand/IOCQ)]

(4 + 2) + (4 + 2) = 12

Group - E

8. (a) Fig. 4 shows a quadrilateral element in global coordinates. Show that the mapping correctly describes the line connecting nodes 2 and 3 and determine the (x, y) coordinates corresponding to $(\xi, \eta,) = (0.3, 0.6)$. [(CO5)(Analysis/IOCQ)]

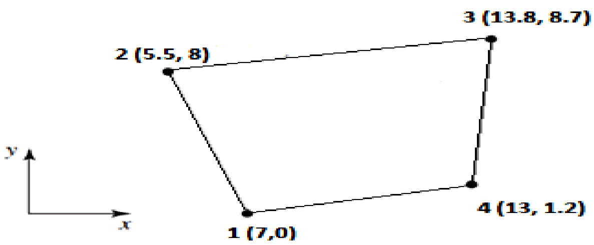

Fig. 4

(b) Evaluate the following integral using two-point and three-point Gaussian quadrature. [(CO5)(Estimate/IOCQ)]

$$\int_{-2}^{8}\int_{2}^{10}(y^3 + 3y^2 + 5y + 2x^2 + x + 10)dxdy$$

6 + (3 + 3) = 12

9. Describe in detail about Pre-Processing and Post-Processingsteps used in a FEA software. Your answer should be accompanied with various schematic representations. [(CO6)(Remember/IOCQ)]

(6 + 6) = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	10.42	64.58	25

Course Outcome (CO):

On completion of this course students will be able to

1. Choose suitable material of a product to be designed as per the application and strength requirement.
2. Relate relevant 'Mode of Failure' and 'Theory of Failure' when solving a problem regarding design of machine components under different types of loadings and boundary conditions.
3. Identify proper stress intensity factors for objects with dimensional discontinuity subjected to different loadings and boundary conditions.
4. Analyse life of a machine component with or without dimensional discontinuity subjected to various dynamic loadings constrained with different boundary conditions.
5. Evaluate detailed specifications for fasteners like screw, nut-n-bolt, for welding and power screw by analysing the machine component subjected to various loading and boundary conditions.
6. Design a solid and hollow shaft, coil and leaf spring, shaft couplings and various belts for a belt drive for given power rating, loadings and boundary conditions.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question