DISCRETE MATHEMATICS
(INFO 3133)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and <u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) $p \vee (p \wedge q) \equiv$
(a) p (b) q
(c) $p \wedge q$ (d) $p \vee q$.

(ii) If $p \leftrightarrow q \equiv (p \to q) \wedge r$ then r is
(a) $p \to q$ (b) $\sim p$
(c) $q \to p$ (d) $\sim q$

(iii) The contrapositive of $\sim p \to q$ is
(a) $p \to q$ (b) $\sim q \to \sim p$
(c) $\sim q \to p$ (d) $q \to \sim p$

(iv) Let a be any positive integer. The number $a(a+1)(a+2)$ is divisible by
(a) 5 (b) 7
(c) 6 (d) 11.

(v) The linear equation $33x + 6y = 8$ has no integral solutions because $\gcd(33,6) = 3$ and
(a) 8 is not divisible by 3 (b) 33 is divisible by 3
(c) 6 is divisible by 3 (d) 33 is not divisible by 6

(vi) Everybody in a room shakes hands with everybody else. The total number of handshakes is 66. The total number of persons in the room is
(a) 11 (b) 12
(c) 13 (d) 14.

(vii) There are 10 lamps in a hall. Each one of them can be switched on independently of the others. The number of ways in which the hall can be illuminated is
(a) 100 (b) 1023
(c) 2 (d) 10!

(viii) The chromatic number of a cycle with n vertices, where n is even, is
(a) 2 (b) 3
(c) 4 (d) 5.

(ix) Determine which of the following statements is not true.
(a) The Petersen graph is non-planar
(b) K_5 is non-planar.
(c) $K_{3,3}$ is non-planar.
(d) The dual graph of a planar graph is non-planar.

(x) For any graph G, if $\omega(G)$ = clique number of G and if $\chi(G)$ = chromatic number of G, then
(a) $\omega(G) = \chi(G)$ (b) $\omega(G) = \chi(G) + 3$
(c) $\omega(G) \le \chi(G)$ (d) $\omega(G) \ge \chi(G)$.

Group – B

2. (a) Prove the following without constructing the truth table.
$(q \to (p \wedge \sim p)) \to (r \to (p \wedge \sim p)) \equiv r \to q$

(b) Determine which of the following compound propositions is a tautology and which is a contradiction, using truth tables.
(i) $\sim (q \to r) \wedge r \wedge (p \to q)$,
(ii) $(\sim q \wedge (p \to q)) \to \sim p$.

4 + (4 + 4) = 12

3. (a) Obtain the principal disjunctive normal form and the principal conjunctive normal form (PDNF and PCNF) of the following. $(\sim p \vee \sim q) \to (p \leftrightarrow \sim q)$

(b) Check the validity of the following argument:
"If a number is divisible by 6 then it is divisible by 3. The number is not divisible by 3. Therefore the number is not divisible by 6."

6 + 6 = 12

Group – C

4. (a) Show that $5^{302} + 3^{603} + 6! \equiv 2 \ (mod\ 7)$. Show all your calculations in detail and state every theorem that you use.

(b) Let p be a prime. If $a \equiv b \ (mod\ p)$, then prove that $a^k \equiv b^k \ (mod\ p)$ for every positive integer k. Is it true that if $a^k \equiv b^k \ (mod\ p)$, then $a \equiv b \ (mod\ p)$? Justify your answer.

6 + 6 = 12

5. (a) Find the general solution of the equation $5x + 32y = 28$ in integers. Express your solution in terms of a parameter t.

(b) Prove the following theorem. There is an infinite number of prime numbers.

6 + 6 = 12

Group – D

6. (a) (i) Assuming that repetitions are not permitted, how many four-digit numbers can be formed from the six digits 1, 2, 3, 5, 7, 8 ?
(ii) How many of the numbers in part (i) are less than 4000 ?
(iii) How many of the numbers in part (i) are even?
(iv) How many of the numbers in part (i) are odd?
(v) How many of the numbers in part (i) are multiples of 5 ?
(vi) How many of the numbers in part (i) contain both the digits 3 and 5 ?

(b) 5 balls are to be placed in 3 boxes. Each can hold all the 5 balls. In how many different ways can we place the balls so that no box is left empty if
(i) balls and boxes are different
(ii) balls are identical and boxes are different
(iii) balls are different and boxes are identical
(iv) balls as well as boxes are identical ?

(6 × 1) + (2 + 2 + 1 + 1) = 12

7. (a) Five gentlemen A, B, C, D and E attend a party, where before joining the party, they leave their overcoats in a cloak room. After the party, the overcoats get mixed up and are returned to the gentlemen in a random manner. Using the principle of inclusion-exclusion, find the probability that none receives his own overcoat.

(b) Use the method of generating functions to solve the recurrence relation
$a_{n+1} - 8a_n + 16a_{n-1} = 4^n$; $n \geq 1$.

5 + 7 = 12

Group – E

8. (a) Prove that the chromatic polynomial $P(G;\lambda)$ of a graph G has degree $n(G)$ with integer coefficients alternating in sign and beginning with $1, -e(G), \ldots$

(b) Prove that $K_{3,3}$ is non-planar.

(c) Find the chromatic number of the following graph (Fig.1):

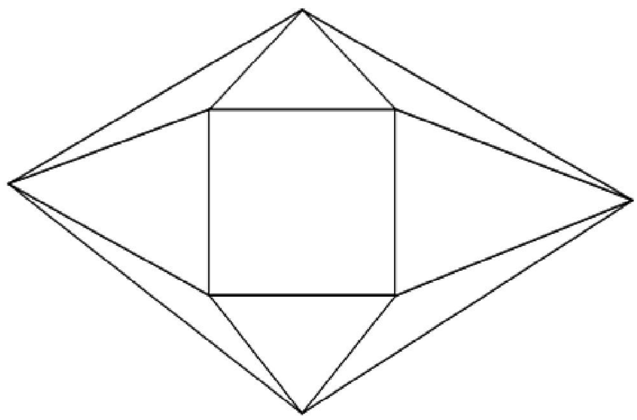

Fig.1

Prove that every regular bipartite graph has a perfect matching.

6 + 4 + 2 = 12

9. Define chromatic number and clique number. Find the chromatic number and clique number of the following graph: (Fig.2)

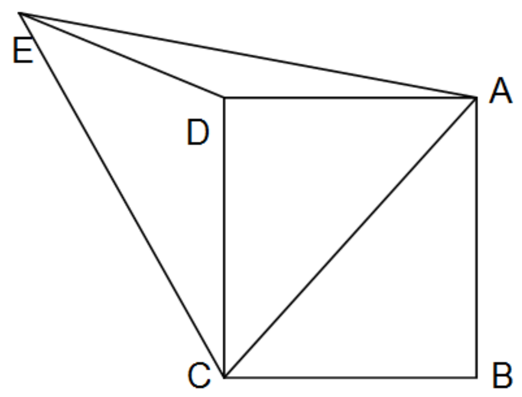

Fig.2

6 + (2 + 2 + 1 + 1) = 12

Department & Section	Submission Link
IT	https://classroom.google.com/c/MjIxMzUwNTA2Nzk0/a/Mjc0NjI4MTY5NTIz/details