# DISCRETE MATHEMATICS
## (CSEN 2102)

**Time Allotted : 3 hrs** **Full Marks : 70**

***Figures out of the right margin indicate full marks.***

***Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.***

***Candidates are required to give answer in their own words as far as practicable.***

## Group – A
## (Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The chromatic number of a cycle having $27$ vertices is
(a) $27$ (b) $28$ (c) $3$ (d) $2$.

(ii) Which one of the following graphs is non-planar?
(a) $K_4$ (b) $K_3$ (c) $K_{2,2}$ (d) $K_{4,4}$.

(iii) $3*18! \equiv$
(a) $16\,(\text{mod}19)$ (b) $15\,(\text{mod}19)$ (c) $18\,(\text{mod}19)$ (d) $14\,(\text{mod}19)$.

(iv) The square of any odd integer is of the form
(a) $4k+3$ (b) $8k+1$ (c) $8k+3$ (d) $8k+5$.

(v) If $4a \equiv 4b\,(\text{mod}\,6)$, then
(a) $a \equiv b\,(\text{mod}\,6)$ (b) $a \equiv b\,(\text{mod}\,2)$
(c) $2a \equiv 2b\,(\text{mod}\,3)$ (d) $5a \equiv 5b\,(\text{mod}\,6)$.

(vi) Three persons enter a railway compartment. If there are $5$ seats vacant in how many ways can they take these seats?
(a) $60$ (b) $20$ (c) $15$ (d) $125$.

(vii) $(p \wedge p) \wedge (p \to (q \wedge q))$ is equivalent to
(a) $p \to q$ (b) $p \wedge q$ (c) $p \vee q$ (d) $q \to p$.

(viii) Given that $(p \wedge q) \wedge (\sim p \wedge \sim q)$ is false, the truth values of $p$ and $q$ are
(a) both false (b) both true
(c) $p$ true and $q$ false (d) $p$ false and $q$ true.

(ix) The generating function of the sequence $1, 1, 1, 1, \ldots$ is given by
(a) $(1-x)^{-1}$ (b) $(1+x)^{-1}$ (c) $(1-x)^{-2}$ (d) $(1+x)^{-1}$.

(x) Total number of seven-letter palindromes formed from English alphabets is
(a) $26^4$ (b) $26^3$ (c) $26^5$ (d) $26^2$.

## Group – B

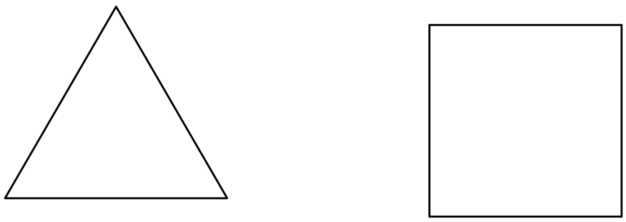
2. (a) Prove that (i) $T_5$, a tree having 5 vertices is planar and (ii) $K_{4,4}$, the complete bipartite graph whose partite sets each have 4 vertices, is non-planar.

(b) (i) Show that $K_4$, the complete graph having 4 vertices satisfies Euler's formula: $f = e - n + 2$, where $n, e, f$ denote the number of vertices, the number of edges and the number of regions of the graph.
(ii) Draw the dual of $C_4$, the square graph having 4 vertices and 4 edges.

**(3 + 3) + (3 + 3) = 12**

3. (a) Find the chromatic polynomial of $K_5$, the complete graph having 5 vertices. Show your work in detail.

(b) Find the chromatic number of (i) $T_6$, a tree having 6 vertices and (ii) the following graph having two components



Justify your answers.

**6 + (2 + 4) = 12**

## Group – C

4. (a) Use Fermat's Little Theorem and Wilson's Theorem to find the remainder in the division of $4^{34} + 3^{722} + 16!$ by 17. Show your calculations and state the theorems.

(b) Let $a \equiv b(\bmod m), c \equiv d(\bmod m)$. Prove that (i) $a + c \equiv b + d(\bmod m)$ and (ii) $ac \equiv bd(\bmod m)$.

**6 + 6 = 12**

5. (a) Find the greatest common divisor of 87 and 32 by using the Euclidean algorithm and express it in the form $87x + 32y$, where $x$ and $y$ are integers.

(b) (i) Solve the equation $17x + 12y = 1$ in integers
(ii) Is the equation $9x + 6y = 2$ solvable in integers? Justify your answer.

**6 + (4 + 2) = 12**

## Group – D

6. (a) Determine the number of integer solutions of the equation $x_1 + x_2 + x_3 + x_4 = 32$

(i) where $x_1, x_2 \geq 5,\ x_3, x_4 \leq 7$,

(ii) and where $x_1, x_2, x_3 \geq 0,\ 0 < x_4 \leq 25$.

(b) Show that the number of derangements of a set of $n$ elements is given by $D_n = n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} \ldots + (-1)^n \frac{1}{n!}\right]$.

**6 + 6 = 12**

7. (a) If $(n+1)$ integers not exceeding $2n$ are selected, show that there must be an integer that divides one of the other integers. Deduce that if 151 integers are selected from $\{1, 2, 3, \ldots, 300\}$ then the selection must include two integers $x, y$ either of which divides the other.

(b) Use the method of generating function to solve the following recurrence relation $a_{n+1} + 4a_n + 4a_{n-1} = n - 1,\ n \geq 1,$ given $a_0 = 0,\ a_1 = 1$.

**6 + 6 = 12**

## Group – E

8. (a) Construct the truth table for the following compound proposition $((p \to q) \to r) \to s$.

(b) Without using truth table, prove the following $p \to (q \to p) \equiv {\sim} p \to (p \to q)$ where $\sim$ denotes the negation of the proposition.

**6 + 6 = 12**

9. (a) Without constructing the truth table find the disjunctive normal form of the following statement $p \wedge {\sim}(q \wedge r) \vee (p \to q)$ where $\sim$ denotes the negation of the proposition.

(b) Without constructing the truth table find the principal conjunctive normal form of the following statement $(p \wedge q) \vee ({\sim} p \wedge q \wedge r)$ where $\sim$ denotes the negation of the proposition.

**6 + 6 = 12**

| Department & Section | Submission Link |
|---|---|
| CSE A | https://classroom.google.com/c/MTQxNzMxMDA2NTEy/a/Mjc0NDQzNzI0NjM1/details |
| CSE B | https://classroom.google.com/c/MTQxNzMxMDA2NTU4/a/Mjc0NDQzNzI0Njgw/details |
| CSE C | https://classroom.google.com/c/MTIxOTY3MTgzMDgx/a/Mjc0NDM0NzEzMDY3/details |
| IT | https://classroom.google.com/c/MTIxOTY3MTgzMDk3/a/Mjc0NDM0NzEzMDkw/details |
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