OPTIMIZATION TECHNIQUES
(MATH 6121)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The number of basic variables in an $m \times n$ transportation problem, is at most
(a) $m+n-1$ (b) $m \times n$ (c) $m-n$ (d) $n-m$.

(ii) If the primal problem and the dual problem both have feasible solution then
(a) dual objective function is unbounded
(b) primal objective function is unbounded
(c) finite optimal for both exists
(d) no solution exists.

(iii) The quadratic form $Q(x,y) = 4x^2 + 2xy - 3y^2$ is
(a) indefinite (b) positive definite
(c) negative semi definite (d) positive semi definite.

(iv) A game is said to be a fair game if the maximin and minimax values of the game are
(a) equal (b) both equal to one
(c) both equal to zero (d) not equal.

(v) Which of the following Hessian matrices belongs to a convex function?
(a) $\begin{pmatrix} -2 & x \\ 0 & -x^2 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 2 \\ 1 & x^2 \end{pmatrix}$ (c) $\begin{pmatrix} -x^2 & x \\ 0 & -x \end{pmatrix}$ (d) $\begin{pmatrix} 0 & -x \\ x & 1 \end{pmatrix}$.

(vi) Given the optimization problem
Optimize $f\ (x,y,z)$ subject to $g(x,y,z) = b$;
it is known that the determinant of last two principal minors of the bordered Hessian matrix $\mathrm{H^B}$ of the Lagrange function $(\mathrm{L(x,y,z,\lambda)})$ at stationary point $(\mathrm{x^*,y^*,z^*;\lambda^*}) = (1,1,1;2)$ is given by
$$H_3 = -12,\ \ H_4 = -20.$$
Then the stationary point $(1,1,1;2)$ is
(a) a maximum point (b) a minimum point
(c) a point of inflection (d) none of these.

(vii) The number of basic variables in a transportation problem of size 7×8 is at most
(a) 13 (b) 14 (c) 15 (d) 16.

(viii) The local maximum of the function $f(x)$ occurs at $x = x_0$ provided
(a) $f^{(n)}(x_0) < 0$, for n odd (b) $f^{(n)}(x_0) > 0$, for n odd
(c) $f^{(n)}(x_0) > 0$, for n even (d) $f^{(n)}(x_0) < 0$, for n even.
(where $f^{(n)}(x_0)$ denotes the n^{th} order derivative of the function $f(x)$ at $x = x_0$)

(ix) The bordered Hessian matrix of the Lagrange function L constructed for an optimization problem with equality constraints is given by

$H^B(L) = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 4 & 0 & 0 \\ 1 & 0 & 4 & 0 \\ 1 & 0 & 0 & 4 \end{pmatrix}$, the number of variables involved in the objective function are

(a) 1 (b) 2 (c) 3 (d) 0.

(x) In an assignment problem, if k be the maximum number of zeros which can be assigned, then the number of lines which will cover all the zeros is

(a) k (b) $2k$ (c) $3k$ (d) $\frac{k}{2}$.

Group – B

2. (a) A person requires 10, 12 and 12 units of chemicals A, B and C respectively for his garden. A liquid product contains 5, 2 and 1 units of A, B and C respectively per jar. A dry product contains 1, 2 and 4 units of A, B and C per carton. If the cost of the liquid product is Rs.30 per jar and that of the dry product is Rs. 20 per carton, find graphically how many of each should he purchase to minimize the cost and meet the requirements. [(MATH6121.1, MATH6121.2)(Create/HOCQ)]

(b) Use simplex method to solve the following L.P.P.:

Maximize $z = 3x_1 + 2x_2$

subject to the constraints

$x_1 + x_2 \leq 4$

$x_1 - x_2 \leq 2$

$x_1, \ x_2 \geq 0.$ [(MATH6121.1, MATH6121.2)(Apply/IOCQ)]

6 + 6 = 12

3. (a) Use Big-M method to solve the following L.P.P.:

Minimize $z = 4x_1 + 3x_2$

subject to the constraints

$x_1 + 2x_2 \geq 8$

$3x_1 + 2x_2 \geq 12$

$x_1, \ x_2 \geq 0.$ [(MATH6121.1, MATH6121.2)(Apply/IOCQ)]

(b) Obtain the dual of the following LPP:

Maximize $z = 6x_1 + 5x_2 + 3x_3$

subject to

$x_1 + x_2 + x_3 \leq 5$

$x_2 + x_3 \geq \frac{3}{5}$

$x_1 + x_2 - x_3 = -1$

$x_1, x_2, x_3 \geq 0$ [(MATH6121.1, MATH6121.2)(Understand/LOCQ)]

7 + 5 = 12

Group – C

4. (a) Solve the following transportation problem by Vogel's approximation method and checking its optimality, find the optimal solution:

	D_1	D_2	D_3	Supply
O_1	26	23	10	61
O_2	14	13	21	49
O_3	16	17	29	90
Demand	52	68	80	

[(MATH6121.1, MATH6121.2, MATH6121.3)(Evaluate/HOCQ)]

(b) Find the optimal assignment and the minimum cost for the assignment with the given cost matrix: [(MATH6121.1, MATH6121.2, MATH6121.3)(Evaluate/HOCQ)]

	1	2	3	4	5
A	6	5	8	11	16
B	1	13	16	1	10
C	16	11	8	8	8
D	9	14	12	10	16
E	10	13	11	8	16

7 + 5 = 12

5. (a) Find the minimum cost solution for the 4×4 assignment problem whose cost coefficients are given below: [(MATH6121.1, MATH6121.2, MATH4121.3)(Evaluate/HOCQ)]

	1	**2**	**3**	**4**
A	4	5	3	2
B	1	4	-2	3
C	4	2	1	-5

(b) Write the mathematical formulation of a transportation problem having m origins and n destinations. [(MATH6121.1, MATH6121.2, MATH4121.3)(Understand/LOCQ)]

6 + 6 = 12

Group – D

6. (a) Use graphical method in solving the following game and hence find the value of the game:

PLAYER B

PLAYER A

2	−1	5	−2	6
−2	4	−3	1	0

[(MATH6121.1, MATH6121.4)(Understand/LOCQ)]

(b) Use dominance to reduce the following pay-off matrix to a 2×2 game and hence find the optimal strategies and the value of the game: [(MATH6121.1, MATH6121.4)(Apply/IOCQ)]

PLAYER B

PLAYER A

3	-2	4
-1	4	2
2	2	6

6 + 6 = 12

7. (a) Use algebraic method to solve the following game:

PLAYER B

PLAYER A

3	−1	−3
−3	3	−1
−4	−3	3

[(MATH6121.1, MATH6121.4) (Apply/IOCQ)]

(b) In a rectangular game, the pay-off matrix is given by:

PLAYER B

PLAYER A

−2	0	0	5	3
4	2	1	2	5
−4	−3	0	−3	6
5	1	−5	2	−6

Find the optimal strategies and the value of the game.

[(MATH6121.1, MATH6121.4)(Understand/LOCQ)]

7 + 5 = 12

Group – E

8. (a) Use Kuhn-Tucker conditions to solve the following non-linear programming problem:
Maximize $z = 10x_1 - x_1^2 + 10x_2 - x_2^2$
subject to the constraints
$x_1 + x_2 \leq 9$
$x_1 - x_2 \geq 6$
$x_1, x_2 \geq 0.$ [(MATH6121.5, MATH6121.6)(Evaluate/HOCQ)]

(b) Determine the relative maximum and minimum (if any) of the following function:
$$f(x_1, x_2, x_3) = x_1^2 + 2x_2^2 + x_3^2 + x_2x_1 - 2x_2 - 7x_1 + 12$$
[(MATH6121.5, MATH6121.6)(Understand/LOCQ)]

8 + 4 = 12

9. Use the method of Lagrange multipliers to solve the following non-linear programming problem. Does the solution maximize or minimize the objective function?
Optimize $z = x_1^2 + x_2^2 + x_3^2 - 10x_1 - 6x_2 - 4x_3$
subject to the constraint
$x_1 + x_2 + x_3 = 7$
$x_1, x_2, x_3 \geq 0.$ [(MATH6121.5, MATH6121.6)(Evaluate/HOCQ)]

12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	24.62	28.32	47.06

Course Outcome (CO):

After the completion of the course students will be able to:
MATH6121.1 Describe the way of writing mathematical model for real-world optimization problems.
MATH6121.2 Identify Linear Programming Problems and their solution techniques.
MATH6121.3 Categorize Transportation and Assignment problems.
MATH6121.4 Apply the way in which Game Theoretic Models can be useful to a variety of real-world scenarios in economics and in other areas.
MATH6121.5 Convert practical situations into non-linear programming problems.
MATH6121.6 Solve unconstrained and constrained programming problems using analytical techniques.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.