ADVANCED DISCRETE MATHEMATICS AND STATISTICAL METHODS
(MATH 5101)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) Three numbers are chosen at random from 1 to 20. The probability that they are consecutive is

(a) $\frac{5}{190}$ (b) $\frac{1}{190}$ (c) $\frac{1}{120}$ (d) $\frac{3}{190}$.

(ii) In a probability distribution of a random variable X the sum of the probabilities is always

(a) 0 (b) less than 1 (c) 1 (d) greater than 1.

(iii) The mean of the Binomial distribution $B(10, \frac{2}{5})$ is

(a) 4 (b) 6 (c) 5 (d) 0.

(iv) Four persons enter a railway carriage in which there are six seats, in how many ways can they take their places?

(a) 720 (b) 360 (c) 180 (d) 24.

(v) From 12 books in how many ways can a selection of 5 be made when one specific book is always included?

(a) 462 (b) 360 (c) 495 (d) 330.

(vi) If X and Y are independent random variables then

(a) $E(XY) = E(X) + E(Y)$ (b) $E(XY) = E(X) - E(Y)$

(c) $E(XY) = E(X)E(Y)$ (d) $E(XY) = \frac{E(X)}{E(Y)}$.

(vii) The generating function of the sequence $1, -2, 2^2, -2^3, 2^4, -2^5, 2^6, -2^7, 2^8, \ldots \ldots$ is

(a) $\frac{1}{1+2x}$ (b) $\frac{1}{1-2x}$ (c) $\frac{1}{(1+2x)^2}$ (d) $\frac{1}{(1+x)^2}$.

(viii) The chromatic number of Kuratowski's second graph is

(a) 1 (b)2 (c) 3 (d) 4.

(ix) If G^* be a dual of a planar graph G, then corresponding to a pendant edge in G, we get
(a) a self-loop in G^* (b) a pendant edge in G^*
(c) an isolated vertex in G^* (d) two parallel edges in G^*.

(x) The number of edges in a tree with 100 vertices is
(a) 98 (b) 99 (c) 100 (d) 101.

Group – B

2. (a) A box contains twenty tickets of identical appearance, the tickets being numbered $1, 2, 3, \ldots 20$. If 3 tickets are chosen at random, find the probability that the numbers on the drawn tickets are in arithmetic progression.
[(MATH5101.1, MATH5101.2)(Evaluate/HOCQ)]

(b) Two boxes contain respectively 4 white, 2 black and 1 white, 3 black balls. One ball is transferred from the first box into the second, and then one ball is drawn from the latter. It turns out to be black. What is the probability that the transferred ball was white? [(MATH5101.1, MATH5101.2)(Apply/IOCQ)]

6 + 6 = 12

3. (a) A continuous random variable X has the following p.d.f:

$$f(x) = \frac{1}{4}(x+2), -1 < x < 1,$$
$$= 0, \text{ elsewhere.}$$

Find the mean and the variance of X. [(MATH5101.1, MATH5101.2)(Apply/IOCQ)]

(b) X is a discrete random variable having probability mass function:

x	0	1	2	3	4	5	6	7
$P(X=x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$.

(i) Determine the constant k.
(ii) Find $P(X < 6)$
(iii) What will be $P(X \geq 6)$?
[(MATH5101.1, MATH5101.2)(Understand/LOCQ)]

6 + 6 = 12

Group – C

4. (a) A and B play a game in which their chances of winning are in the ratio $3:2$. Find A's chances of winning at least three games out of the five games played.
[(MATH5101.1, MATH5101.2)(Apply/IOCQ)]

(b) In a normal distribution 31% of the items are under 45 and 8% are above 64. Find the mean and standard deviation of the distribution.
[Given: $P(0 < Z < 1.405) = 0.42$, $P(-0.496 < Z < 0) = 0.19$]
[(MATH5101.1, MATH5101.2)(Evaluate/HOCQ)]

6 + 6 = 12

5. (a) Find the mean and the standard deviation of the first n natural numbers.
[(MATH5101.1, MATH5101.2)(Evaluate/HOCQ)]

(b) For two variables x and y the equations of two regression lines are $x+4y+3=0$ and $4x+9y+5=0$. Identify which one is of 'y on x'. Find the means of x and y. Find

the correlation coefficient between x and y. Estimate the value of x when $y = 1.5$.

[(MATH5101.1, MATH5101.2)(Create/HOCQ)]

6 + 6 = 12

Group – D

6. (a) State the Pigeonhole Principle. Use it to prove that if m is an odd positive integer, then there exists a positive integer n such that m divides $2^n - 1$.

[(MATH5101.1, MATH5104.2, MATH5101.3)(Create/HOCQ)]

(b) Suppose that we draw a card from a pack of 52 cards and replace it before the next draw. In how many ways can 10 cards be drawn so that the tenth card is a repetition of a previous draw? [(MATH5101.1, MATH5104.2, MATH5101.3)(Analyze/IOCQ)]

6 + 6 = 12

7. (a) How many integral solutions are there to $x_1 + x_2 + x_3 + x_4 + x_5 = 20$ where $x_1 \geq 3, x_2 \geq 2, x_3 \geq 4, x_4 \geq 6,$ and $x_5 \geq 0$? Show your work in detail.

[(MATH5101.1, MATH5101.2, MATH5101.3)(Evaluate/HOCQ)]

(b) Solve the following recurrence relation by the method of generating functions: $a_n = 8a_{n-1} + 10^{n-1}$, where $a_0 = 1, a_1 = 9$.

[(MATH5101.1, MATH5101.2, MATH5101.3)(Create/HOCQ)]

5 + 7 = 12

Group – E

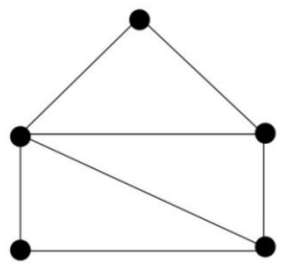
8. (a) Let G be a simple connected planar graph with n_v vertices, n_e edges and n_f regions. Then prove that (i) $n_e \geq \frac{3}{2} n_f$ and (ii) $n_e \leq 3n_v - 6$.

[(MATH5101.1, MATH5101.2, MATH5101.4)(Understand/LOCQ)]

(b) What is a perfect matching? How many perfect matchings are there in a complete graph of 6 vertices? [(MATH5101.1, MATH5101.2, MATH5101.4)(Evaluate/HOCQ)]

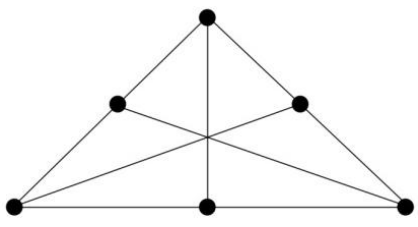
(3 + 3) + (2 + 4) = 12

9. (a) By using the decomposition theorem, determine the chromatic polynomial for the following graph. Hence find its chromatic number.



[(MATH5101.1, MATH5101.2, MATH5101.4)(Evaluate/HOCQ)]

(b) Prove that the following graph is non-planar.



[(MATH5101.1, MATH5101.2, MATH5101.4)(Analyze/IOCQ)]

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	12.5	31.25	56.25

Course Outcome (CO):

After the completion of the course students will be able to
MATH5101.1. To understand the mathematical fundamentals that is prerequisites for a variety of courses like Data mining, Network protocols, analysis of Web traffic, Computer security, Software engineering, Computer architecture, operating systems, distributed systems, Bioinformatics, Machine learning.
MATH5101.2. To develop the understanding of the mathematical and logical basis to many modern techniques in information technology like machine learning, programming language design, and concurrency
MATH5101.3. To study the principles of enumeration
MATH5101.4. To equip oneself with the techniques used in graph theory.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.