COMPUTATIONAL METHODS IN ENGINEERING
(MECH 4128)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The total numerical error is
(a) Round off error + Truncation error
(b) Round off error + 1.5 × Truncation error
(c) Round off error + 3 × Truncation error
(d) Round off error - Truncation error.

(ii) The number of significant digits in 0.051 is
(a) 3 (b) 5 (c) 2 (d) 6.

(iii) Which is not a part of Gauss Elimination process?
(a) Forward elimination (b) Backward substitution
(c) Augmentation of the matrix (d) Inversion of the matrix.

(iv) In Newton Raphson method which order derivative is incorporated in the formula?
(a) First order (b) Second order
(c) Third order (d) Fourth order.

(v) In polynomial regression method, to fit a 3rd order polynomial no of equations required is
(a) 2 (b) 3 (c) 4 (d) 5.

(vi) Lagrange interpolating polynomial is used to find
(a) the root of an equation
(b) the solution of two linear equations
(c) the functional value at a point when a set of data is given
(d) the integral value of a function.

(vii) In Trapezoidal rule, the polynomial is approximated as
(a) linear (b) second order (c) third order (d) fourth order.

(viii) For an n^{th} order differencing scheme, if the grid spacing is halved the (truncation) error would reduce by a factor of
(a) 2^{n-1} (b) 2^{n} (c) 2^{n+1} (d) $2n$.

(ix) Consider the following statements:

(I) An Initial Value Problem (**IVP**) is one in which a solution to a differential equation is obtained subject to conditions on the unknown function and its derivatives $\left(\frac{dy}{dx}, \frac{d^2y}{dx^2}, \ldots, \frac{d^{n-1}y}{dx^{n-1}}\right)$ specified at one value of the independent variable

(II) A Boundary Value Problem (**BVP**) is one in which a solution to a differential equation is obtained subject to conditions on the unknown function and its derivatives $\left(\frac{dy}{dx}, \frac{d^2y}{dx^2}, \ldots, \frac{d^{n-1}y}{dx^{n-1}}\right)$ specified at two or more values of the independent variable

(III) A Well-Posed Problem is one in which the number$(= n)$ of prescribed conditions are exactly same as the order $(= n)$ of the equation.

Out of the three statements above:

(a) only (I) is true (b) only (I) and (II) are true

(c) only (I) and (III) are true (d) (I), (II), and (III) are all true.

(x) Consider the following statements:

(I) Every time-dependent PDE is parabolic in nature.

(II) Parabolic PDEs with two independent variables have a single characteristic curve only.

Which of the above statements is/are correct?

(a) Both (I) and (II) (b) Only (I)

(c) Only (II) (d) Neither (I) and (III).

Group – B

2. (a) A differential equation for a typical parachutist problem is written below where v is velocity (m/s), g is gravitational acceleration, c is coefficient of drag (kg/s), m is mass of the system (kg) t is time (s).

$$\frac{dv}{dt} = g - \frac{c}{m}v$$

Find out the value of v at t = 2, 4, 6 and 8 s by Euler's numerical analysis. The value of c is 14 kg/, m is 72 kg and initial velocity is 0 m/s. [(CO1)(Evaluate/HOCQ)]

(b) Write down the definition of true error and relative error. Explain the significance of relative error. [(CO1)(Remember/LOCQ)]

8 + 4 = 12

3. (a) Use the Bairstow's method to find the root of the function $f(x) = 0.7x^3 - 4x^2 + 6.2x - 2$. [(CO2)(Understand/LOCQ)]

(b) Using Gauss Seidel method, solve the following set of linear equations. The solution must be correct upto 2 decimal places. [(CO2)(Evaluate/HOCQ)]

$$10x + 2y - z = 27$$
$$-3x - 6y + 2z = -61.5$$
$$x + y - 5z = -21.5$$

5 + 7 = 12

Group – C

4. (a) Using polynomial regression, fit a parabola through certain points and find out the equation of the parabola for the following data (the function is z = f(y)):

y	2.5	3.5	4.5	5.5	6.5	7.5
z	18	32	66	78	91	103

[(CO3)(Analyze/IOCQ)]

(b) Write down the expression of the slope and the constant for a linear regression method. [(CO3)(Remember/LOCQ)]

10 + 2 = 12

5. (a) Calculate f(4) from the following data using Newton's interpolating polynomial:

x	1	2	3	5	7	8
y	3	6	19	99	291	444

[(CO3)(Analyze/IOCQ)]

(b) The following data defines the sea level concentration of dissolved oxygen for fresh water as a function of temperature:

T (^{0}C)	0	8	16	24	32	40
C (mg/L)	14.62	11.84	9.87	8.42	7.31	6.41

Estimate C (29^{0}C) using Lagrange interpolating polynomial. [(CO3)(Solve/IOCQ)]

6 + 6 =12

Group – D

6. (a) Using Trapezoidal rule, evaluate the following integral (take n = 4):

$$\int_{-2}^{6}(1+e^x)dx$$ [(CO4)(Solve/IOCQ)]

(b) Evaluate the integral value of following function using 2 point Gauss Quadrature:

$$f(x) = \frac{e^x \cos x}{1+x^2}$$ [(CO4)(Evaluate/HOCQ)]

4 + 8 = 12

7. (a) Use the Taylor series expansion to evaluate the integral of $y' - 5y = e^x;\ y(0) = 0$ at (i) x = 0.1 (0.1) 0.3 (ii) x = 1.0; 1.1 [Retain terms up tox^5] [CO5/Implement/IOCQ]

(b) Evaluate the initial value problem $y' - 5y = 0;\ y(0) = 1$ by the Euler's method at x = 0.1 and x = 0.2. [CO5/Formulate/HOCQ]

(5 + 2) + 5 = 12

Group – E

8. (a) Use the Milne's predictor-corrector method to solve the IVP $y' - 5y = 0$ at x =0.4 and at x = 0.5, given:

$y(0) = 1;\ y(0.1) = 1.382;\ y(0.2) = 2.218;\ y(0.3) = 3.246.$ [CO5/Implement/IOCQ]

(b) Match the following:

(I) $2-$ D Laplace equation $u_{xx} + u_{yy} = 0^1$	(A) Parabolic
(II) $1-$ D wave equation $u_{tt} = c^2 u_{xx}$	(B) Hyperbolic
(III) $1-$ Dtransient heat conduction equation$u_t = c^2 u_{xx}$	(C) Elliptic

[CO6/Judge/HOCQ]

9 + 3 = 12

9. (a) Compute numerically the solution of the one-dimensional wave equation:

$$20\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial t^2};\ 0 \le x \le 5, t > 0.$$

The boundary conditions are as follows:

$$f(x = 0, t) = f(x = 5, t) = 0.$$

The initial conditions are as follows:

$$f(x, t = 0) = 20x; 0 \le x \le 1; f(x, t = 0) = 25\left(1 - \frac{x}{5}\right); 1 \le x \le 5;\ \left.\frac{\partial f}{\partial t}\right|_{t=0} = 0.$$

[CO6/Formulate/HOCQ]

(b) Find the condition for the second order PDE $u_{xx} + 2xu_{xy} + (1 - y^2)u_{yy} = 0$ to be hyperbolic. [CO6/Classify/LOCQ]

9 + 3 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	14.58	43.75	41.67

Course Outcome (CO):

After the completion of the course students will be able to

CO 1: Apply mathematical models for numerical solutions and classify different types of error.

CO 2: Solve a system of linear algebraic equations by different methods and find out the roots.

CO 3: Implement the regression and interpolation methods for curve fitting and solve different types of optimization problems.

CO 4: Use different numerical integration methods for practical problems.

CO 5: Classify Initial and Boundary value problems to select appropriate solution strategies, and solve Eigenvalue problems applied to physical systems.

CO 6: Apply the Finite Difference Method and the Finite Element Method to formulate and develop solutions for one-dimensional and two-dimensional problems in partial differential equations.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.