PHYSICS - II
(PHYS 2101)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) For a two-particle coupled spring-mass system the normal coordinates are given by $q_1 = \frac{1}{\sqrt{2}}(x_1 + x_2)$ and $q_2 = \frac{1}{\sqrt{2}}(x_1 - x_2)$. The symmetric mode corresponds to
(a) $q_1 = 0$ (b) $q_2 = 0$ (c) $q_1 + q_2 = 0$ (d) $q_1 - q_2 = 0$

(ii) For an N-particle system the maximum number of normal frequency is
(a) N (b) 2N (c) N-1 (d) 2N+1

(iii) If H is the Hamiltonian of a system
(a) $\frac{\partial H}{\partial t} + \frac{dH}{dt} = 0$ (b) $\frac{\partial H}{\partial t} - \frac{dH}{dt} = 0$ (c) $\frac{\partial H}{\partial t} + H = 0$ (d) $H + \frac{dH}{dt} = 0$

(iv) The cyclic coordinates in the Lagrangian $L = \frac{1}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - ky$ are
(a) x and y (b) y and z (c) x and z (d) x, y, z

(v) What is the angular momentum vector in an orbital motion?
(a) The vector is perpendicular to the orbital plane
(b) The vector is along the radius vector
(c) The vector is parallel to the linear momentum
(d) The vector is in the orbital plane.

(vi) When the external force on the system is zero then ______ remains constant.
(a) velocity of body (b) velocity of centre of mass
(c) both (a) and (b) (d) none of (a) and (b)

(vii) If the total external torque on a system of particles is zero then
(a) Total linear momentum of the system is conserved
(b) Total angular momentum of the system is conserved
(c) Both (a) and (b)
(d) Either (a) or (b).

(viii) The bulk modulus is expressed in terms of Young's modulus (Y) and Poisson's ratio (ν) as
(a) $\frac{Y}{3(1-2\nu)}$ (b) $\frac{Y}{3(1+2\nu)}$ (c) $\frac{Y}{2(1-3\nu)}$ (d) $\frac{Y}{2(1+3\nu)}$

(ix) The strain tensor corresponding to a plane strain condition has
(a) Nine components of which three are independent
(b) Nine components of which six are independent
(c) Four components of which three are independent
(d) Four components of which two are independent.

(x) What is the formula for the theorem of perpendicular axis?
(a) $2I_{zz} = I_{xx} - I_{yy}$ (b) $I_{zz} = I_{xx} + I_{yy}$
(c) $2I_{zz} = I_{xx} + I_{yy}$ (d) $I_{zz} = 2(I_{xx} + I_{yy})$
Where the symbols have their usual meaning.

Group- B

2. (a) Three equal point masses m are located at $(a,0,0)$, $(0,a,2a)$ and $(0,2a,a)$. Show that the moment of inertia tensor is not a diagonal matrix and find the principal moment of inertia about the origin. [(CO1)(Evaluate/IOCQ)]

(b) A rigid body consists of three point masses of 2 kg, 1 kg, and 4 kg, connected by massless rods. These masses are located at coordinates (1,-1,1), (2,0,2), and (-1,1,0) in meters, respectively. Compute the inertia tensor of this system. What is the angular momentum vector of this body, if it is rotating with an angular velocity $\vec{\omega} = 3\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$ where the symbols have their usual meaning. [(CO1)(Analyse/IOCQ)]

(3 + 3) + (3 + 3) = 12

3. (a) Find the principal moments of inertia at the centre of a uniform rectangular plate (negligible thickness) of sides a and b. [(CO1)(Evaluate/IOCQ)]

(b) (i) Prove that the total angular momentum of a system of particles about any point 'O' equals the angular momentum of the total mass assumed to be located at the center of mass plus the angular momentum of motion about the center of mass. [(CO1)(Understand/LOCQ)]

(ii) Calculate the moment of inertia and products of inertia for rotation about Z axis with angular velocity $\vec{\omega}$, of two particles of equal mass m located at $(0, y_0, z_0)$ and $(0, -y_0, z_0)$. Is angular momentum in the same direction as angular velocity? Discuss. [(CO1)(Analyse/HOCQ)]

6 + (3 + 3) = 12

Group - C

4. (a) An Atwood machine is constructed with an inextensible massless string of length l mounted over a massless pulley and carrying two unequal masses m_1 and m_2. The radius of the pulley is R.
(i) Construct the constraint relations for the system. [(CO3)(Create/HOCQ)]
(ii) Find the degrees of freedom of the system. [(CO3)(Understand/LOCQ)]
(iii) Construct the Lagrangian of the problem. [(CO3)(Create/HOCQ)]
(iv) Construct the Lagrange equation of motion. [(CO3)(Create/HOCQ)]

(b) Show that if L is not an explicit function of time $\frac{\partial L}{\partial \dot{q}}\dot{q} - L = \text{constant}$.
[(CO2)(Remember/LOCQ)]

(3 + 1 + 3 + 2) + 3 = 12

5. (a) The Hamiltonian of a system is given by $H = \frac{p_1{}^2}{2m} + \frac{p_2{}^2}{2mq_1{}^2} + \frac{k}{q_1{}^2}$.
(i) Construct the Hamilton's equations of motion. [(CO3)(Create/HOCQ)]
(ii) Justify the fact that the system is conservative. [(CO3)(Evaluate/HOCQ)]
(b) Find the components of generalized momentum. [(CO3)(Remember/LOCQ)]
(c) Identify the cyclic coordinate (if any). [(CO3)(Apply/IOCQ)]

(4 + 2) + 4 + 2 = 12

Group - D

6. (a) Define SHM in the frame work of Lagrangian Mechanics and show that such a system is conservative. [(CO4)(Remember/LOCQ)]
(b) A system has the potential $V(q_1,\ q_2) = \frac{1}{2}q_1{}^2 + \frac{1}{2}q_2{}^2 - kq_1q_2, k > 0$
(i) Identify the point of equilibrium of the system. [(CO4)(Apply/IOCQ)]
(ii) Determine the value of k for which system can show small oscillation. [(CO4)(Evaluate/HOCQ)]

(2 + 4) + (2 + 4) = 12

7. The Lagrangian of a coupled system is given by $L = \frac{1}{2}\dot{x}_1{}^2 + \frac{1}{2}\dot{x}_2{}^2 - 3(x_1{}^2 + x_2{}^2 - x_1x_2)$.
(i) Evaluate the normal frequencies and normal modes. [(CO4)(Evaluate/HOCQ)]
(ii) Demonstrate that the given Lagrangian can be represented as the sum of Lagrangians of two independent oscillations. [(CO4)(Understand/LOCQ)]

(4 + 4 + 4) = 12

Group – E

8. (a) Derive the expression for the bulk modulus in terms of the Young's modulus and Poisson's ratio for a homogeneous and isotropic elastic body. [(CO6)(Apply/IOCQ)]
(b) Write down the equilibrium equations of a differential element in plane stress using indicial notation. Determine whether the following stress state satisfies equilibrium:
$$\begin{pmatrix} 2x^3y^2 & -2x^2y^3 \\ -2x^2y^3 & xy^4 \end{pmatrix}$$
[(CO6)(Evaluate/HOCQ)]
(c) When is a flow steady? When is a flow incompressible? Explain using relevant equations. [(CO5)(Remember/LOCQ)]
(d) For the velocity field $\vec{V} = (2x^2 - xy + z^2)\hat{\imath} + (x^2 - 4xy + y^2)\hat{\jmath} + (-2xy - yz + y^2)\hat{k}$, find $\frac{1}{\rho}\frac{D\rho}{Dt}$. [(CO5)(Remember/LOCQ)]

3 + (1 + 2) + (2 + 2) + 2 = 12

9. (a) Write down the components of the strain tensor in three dimensions. A displacement field is given as
$$\vec{u} = (x^2 + 3)\hat{\imath} + (3y^2z)\hat{\jmath} + (x + 3z)\hat{k}$$
What are the strain components at the point $(0, 2, 3)$? [(CO6)(Remember/LOCQ)]
(b) Apply the equation of continuity to a flow in a pipe that divides into two exiting areas with different densities at each area assuming uniform flows over all three areas. [(CO5)(Apply/IOCQ)]

(c) An incompressible flow is described by a velocity field $\vec{V} = 4xy^2\hat{\imath} + f(y)\hat{\jmath} - zy^2\hat{k}$. Evaluate the appropriate form of $f(y)$ that satisfies continuity relation.

[(CO5)(Evaluate/HOCQ)]

(2+4) + 3 + 3 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	41.7	26	32.3

Course Outcome (CO):

After the completion of the course students will be able to

1. Understand angular momentum kinetic energy and motion of a rigid body with applications in mechanical systems.
2. Understand calculus of variation as a core principle underlying majority of the physical laws: Newton's laws, Laplace equation (electrostatics and fluid mechanics), wave equation,heat conduction equation, control theory and many other.
3. Appreciate dynamical equations as a consequence of variational extremization of action functional along with the use of Euler-Lagrange equation to understand the behaviour of simple mechanical systems.
4. Appreciate the ubiquity of oscillation physics-from pendulum and spring-mass system to electrical circuit and movement of piston and comprehend the small motion of a system around stable equilibrium through the notion of normal modes—the meaning of eigen value problem in oscillation physics.
5. Elucidate the basic principles of fluid mechanics through the study of mass conservation, momentum balance, and energy conservation applied to fluids in motion.
6. Understand the mechanics of deformable bodies through a study of the concepts of normal and shear stresses and strains, following a review of the principles of statics.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.