DIGITAL SIGNAL PROCESSING
(ELEC 3141)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The sampling frequency of the signal $x(t) = 4\sin(300\pi t) + 2\cos(50\pi t)$ should be
(a) greater than 300 Hz (b) greater than 150 Hz
(c) lesser or equal to 150 Hz (d) greater or equal to 25 Hz.

(ii) If a signal $x(n)$ having N samples is convoluted with $h(n)$ having M samples then the after linear convolution the obtained signal will have
(a) (M+N) samples (b) (M+N-1) samples
(c) (M-N) samples (d) (M-N+1) samples.

(iii) The structure that uses separate delays for input and output samples is
(a) direct form-II (b) direct form-I
(c) cascade (d) parallel.

(iv) The R.O.C of $z-transform$ of the discrete signal $x(n) = 2^n u(n)$ is
(a) R.O.C: $|z| < 2$ (b) R.O.C: Complete $z-complex$ plane
(c) R.O.C: $|z| > 2$ (d) R.O.C= inside the unit circle of $z-plane$.

(v) In bilinear transformation the analog system with transfer function $H(s) = \frac{0.2}{s+0.9}$ is transformed to a digital system with transfer function,
(a) $H(z) = \frac{0.2}{\frac{2}{T}\frac{1+z^{-1}}{1-z^{-1}}+0.9}$ (b) $H(z) = \frac{0.2}{\frac{T(1-z^{-1})}{2(1+z^{-1})}+0.9}$
(c) $H(z) = \frac{0.2}{\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}+0.9}$ (d) $H(z) = \frac{0.2}{\frac{T(1+z^{-1})}{2(1-z^{-1})}+0.9}$.

(vi) The DFT coefficient $X(1)$ of the four point segment $x(0) = 1,\ x(1) = 0,\ x(2) = 0,\ x(3) = 1$ of a sequence $x(n)$ is
(a) $X(1) = 0$ (b) $X(1) = 1 - j$ (c) $X(1) = 1 + j$ (d) $X(1) = 1 + j2$.

(vii) If ω_c is the cut-off frequency of the low pass filter, then the response lies only in the range
(a) $-\omega_c \le \omega \le \pi$ (b) $-\omega_c \le \omega \le \omega_c$ (c) $-\pi \le \omega \le -\omega_c$ (d) $\omega_c \le \omega \le \pi$.

(viii) Let $x(n) = \{1,\ 2,\ \ 0,\ \ 3\}$ for $n = 0\ to\ 3$. The circularly folded signal $x(-n)$ is
(a) $\{1,\ 3,\ 0,\ 2\}$; for $n = 0\ to\ 3$ (b) $\{1,\ 3,\ 0,\ 2\}$; for $n = 0\ to\ 3$
(c) $\{3,\ 0,\ 1,\ 2\}$; for $n = 0\ to\ 3$ (d) $\{0,\ 3,\ 1, 2\}$; for $n = 0\ to\ 3$.

(ix) The deviation of the group delay from a constant indicates the degree of
(a) linearity of the phase (b) symmetry of the phase
(c) non-linearity of the phase (d) non-symmetry of the phase.

(x) The width of the main-lobe in rectangular window spectrum is
(a) $\frac{4\pi}{N}$ (b) $\frac{16\pi}{N}$ (c) $\frac{8\pi}{N}$ (d) $\frac{2\pi}{N}$.

Group – B

2. (a) Explain aliasing phenomenon with a proper example. What do you mean by Nyquist rate? [(CO1)(Remember/LOCQ)]

(b) Examine whether the following signal $g(n)$ is an energy or a power signal.

$$g(n) = \left(\frac{1}{3}\right)^n u(n)$$ [(CO1)(Analyse/IOCQ)]

(c) Determine $y(n) = x\left(\frac{n}{2} + 2\right)$ and $z(n) = x\left(n - \frac{1}{2}\right)$, when $x(n) = \left\{2, \overset{\downarrow}{\underset{\frown}{1}}, 4, 6\right\}$.

[(CO1)(Analyse/IOCQ)]

(3 + 1) + 3 + (2 + 3) = 12

3. (a) Evaluate the output of the system whose impulse response is given by $h(n) = \left\{2, 5, \overset{\downarrow}{\underset{\frown}{0}}, 4\right\}$ for an input $x(n) = \left\{4, \overset{\downarrow}{\underset{\frown}{1}}, 3, \right\}$ by convolution technique.

[(CO1)(Evaluate/HOCQ)]

(b) Examine whether the following system is a linear or not.

$$y(n) = 2x(n) + \frac{1}{x(n-2)}.$$ [(CO1)(Analyse/IOCQ)]

(c) Examine whether the following system is time invariant or not.

$$y(n) = x(-n - 2).$$ [(CO1)(Analyse/IOCQ)]

(d) Solve the difference equation described by,

$$C(k + 2) - 5C(k + 1) + 2C(k) = u(k)$$

Given that $C(0) = 0$, $C(1) = 2$. Consider $C(k)$ as output and $u(k)$ as input to the system which is a unit step signal. [(CO1)(Analyse/IOCQ)]

4 + 2 + 2 + 4 = 12

Group – C

4. (a) Determine the z-transform and ROC of the discrete time signal

$$x(n) = (0.3)^n\ u(n) + (0.8)^n\ u(-n - 1)$$

[(CO2)(Analyze/IOCQ)]

(b) Determine the inverse z-transform of $X(z) = \frac{1}{1 - 4.5z^{-1} + 3.5z^{-2}}$, if ROC $|z| > 3.5$.

[(CO2)(Analyze/IOCQ)]

(c) Evaluate the impulse response $h(n)$ for the system described by the 2nd order difference equation $y(n) - 4y(n-1) + 3y(n-2) = x(n) + 2x(n-1)$, where $x(n)$ and $y(n)$ are the input and output sequence of the system.

[(CO2)(Evaluate/HOCQ)]

4 + 4 + 4 = 12

5. (a) Derive the transformation formula for bilinear transformation. What is frequency warping? [(CO2)(Understand/LOCQ)]

(b) Transform the analog filter having a transfer function $H(s) = \frac{1}{(s+2)(s+3)}$, into a digital filter using bilinear transformation for sampling time, $T = 1$sec.

[(CO2)(Analyze/IOCQ)]

(c) Evaluate the digital system transfer function $H(z)$ for the analog transfer function $H(s) = \frac{1}{s^2+\sqrt{2}s+1}$ using backward difference methods for sampling time, $T = 0.1$sec.

[(CO2)(Evaluate/HOCQ)]

(3 + 1) + 4 + 4 = 12

Group – D

6. (a) Determine the DTFT of the sequence $(n) = a^{|n|}; a < 1$. [(CO3)(Analyze/IOCQ)]

(b) Find the inverse DTFT of $X(e^{j\omega}) = 2\pi\delta(\omega - \omega_0);\ |\omega_0| \leq \pi$. [(CO3)(Analyze/IOCQ)]

(c) Evaluate 4-point DFT of the sequence $x(n) = cos(\frac{n\pi}{4})$. [(CO3)(Evaluate/HOCQ)]

(d) Evaluate inverse DFT of $X(k) = \{3, (2+j), 1, (2-j)\}$ using matrix method.

[(CO3)(Evaluate/HOCQ)]

4 + 2 + 3 + 3 = 12

7. (a) An 8-point sequence is given by $x(n) = \{2,1,2,1,1,2,1,2\}$. Compute 8-point DFT of $x(n)$ using DIT-FFT algorithm. [(CO3)(Evaluate/HOCQ)]

(b) Calculate the percentage saving in calculation of a 512-point radix-2 FFT, when compared to direct DFT. [(CO3)(Understand/LOCQ)]

9 + 3 = 12

Group – E

8. Obtain the direct form-I, direct form-II and cascade form realization of the LTI system governed by the equation

$$y(n) = -\frac{3}{4}y(n-1) + \frac{1}{2}y(n-2) + \frac{1}{4}y(n-3) + x(n) + 4x(n-1) + 3x(n-2).$$

[(CO4)(Analyze/IOCQ)]

4 + 4 + 4 = 12

9. (a) What are FIR filters? Describe the advantages of using FIR filters over the IIR filters.

[(CO4)(Remember/LOCQ)]

(b) Design a linear phase FIR high pass filter with a cut-off frequency, $\omega_c = 0.8\pi$ rad/samples using 7 samples of Hamming window sequence.

[(CO4)(Evaluate/HOCQ)]

(2 + 2) + 8 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	15	46	39

Course Outcome (CO):

After the completion of the course students will be able to

CO1: Understand, interpret, represent, manipulate, process, and analyze of discrete time signals and systems in the context of digital signal processing.

CO2: Understand a new representation of signal sequences with the z-transform, concept of transfer-function, and an application of z-transform properties for modeling of discrete time signals and stability analysis of systems.

CO3: Understand the frequency domain analysis of discrete time signals, spectral analysis and existence of efficient and fast algorithms for DSP systems.

CO4: Understand the design and analyze for digital filters, concept of linear-phase filters, realization of filter structures, mapping from analog filter to digital filter, and implementation of digital filters in real time (with Digital signal processor).

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question