DISCRETE MATHEMATICS
(MATH 2103)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and <u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) For a non-null bipartite graph G which one of the following is true?
(a) G must contain a circuit of length 3
(b) G contains no circuit of length 3
(c) G may contain a circuit of length 3
(d) G must contain a circuit of length 4.

(ii) If a, b, c are positive integer such that $\gcd(a, b) = 1$ and $a|bc$, then
(a) $a|b$ (b) $b|c$ (c) $a|c$ (d) $c|a$.

(iii) The integer 3 belongs to the same residue class of integer modulo 5 as
(a) -6 (b) -7 (c) -8 (d) -9.

(iv) 5 Books are arranged in alphabetical order by author's name. In how many ways can we re-arrange the books so that no book is in its original positions?
(a) 45 (b) 44 (c) 24 (d) 120.

(v) Which one of the following proposition is a tautology?
(a) $(p \lor q) \to p$ (b) $p \lor (q \to p)$
(c) $p \lor (p \to q)$ (d) $p \to (q \to p)$.

(vi) A connected planar graph with 8 vertices determines 4 regions. The number of edges of this graph is
(a) 4 (b) 8 (c) 10 (d) 12.

(vii) Negations of $\exists x\ \forall y,\ P(x, y)$ is
(a) $\forall x\ \forall y,\ P(x, y)$ (b) $\exists x\ \exists y,\ P(x, y)$
(c) $\exists x\ \exists y,\ \sim P(x, y)$ (d) $\forall x\ \exists y,\ \sim P(x, y)$.

(viii) The chromatic polynomial of a null graph of order n, $P_n(\lambda)$, is given by
(a) λ^n (b) λ^{n-1} (c) $\lambda(\lambda - 1)^{n-1}$ (d) $\lambda(\lambda - 1)^n$.

(ix) The highest power of 3 which is contained in 15! is
(a) 6 (b) 7 (c) 8 (d) 9.

(x) The generating function for the sequence $1, 3, 3^2, 3^3, \ldots$ (given $|3x| < 1$) is
(a) $\frac{1}{(1-3x)^2}$ (b) $\frac{1}{1-3x}$ (c) $\frac{3}{1-3x}$ (d) $\frac{x}{1-3x}$.

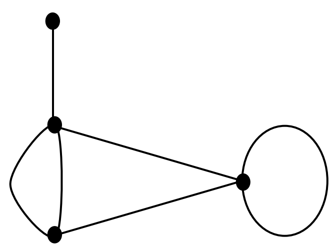
Group – B

2. (a) In how many ways a tree with 5 vertices can be coloured with at most 4 colours?
[(MATH2103.1, MATH2103.2)(Analyze/IOCQ)]

(b) Mention all the reasons why the polynomial $\lambda^5 - 3\lambda^4 - 2\lambda^3 - \lambda^2$ cannot be a chromatic polynomial of a connected graph.
[(MATH2103.1, MATH2103.2)(Analyze/IOCQ)]

(c) Let G be a simple planar graph with less than 12 vertices. Prove that G has a vertex whose degree is less than equals to 4.
[(MATH2103.1, MATH2103.2)(Analyze/IOCQ)]

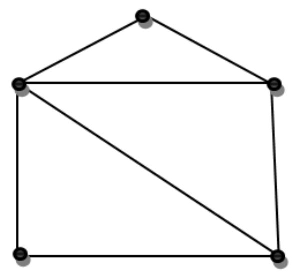
(d) Draw the dual of the following graph:



[(MATH2103.1, MATH2103.2)(Understand/LOCQ)]

3 + 3 + 3 + 3 = 12

3. (a) Use decomposition theorem to find the chromatic polynomial of the following graph and hence find the chromatic number.



[(MATH2103.1, MATH2103.2)(Apply/IOCQ)]

(b) Define maximal matching, maximum matching and perfect matching of a graph. Find one maximal matching, one maximum matching and the matching number for the graph C_7, the cycle graph with seven vertices.
[(MATH2103.1, MATH2103.2) (Remember/LOCQ)]

6 + 6 = 12

Group – C

4. (a) Use the theory of congruences to find the remainder when $2^{73} + 14^3$ is divided by 11.
[(MATH2103.3)(Analyze/IOCQ)]

(b) Apply the Euclidean algorithm to find all possible solutions of the Diophantine equation $37x + 23y = 1$. [(MATH2103.3)(Apply/IOCQ)]

6 + 6 = 12

5. (a) State the Well-Ordering Principle. [(MATH2103.3)(Remember/LOCQ)]

(b) Find the remainder when the sum $1! + 2! + 3! + \cdots + 500!$ is divided by 5.
[(MATH2103.3)(Understand/LOCQ)]

(c) Show that $3^{204} + 4^{302} + 10! \equiv 8 \pmod{11}$. State every theorem that you use. Show your calculations in detail. [(MATH2103.3) (Analyze/IOCQ)]

1 + 4 + 7 = 12

Group – D

6. (a) There are 21 consonants and 5 vowels in the English alphabet. Consider only 8-letter words with 3 different vowels and 5 different consonants.
   (i) How many such words can be formed?
   (ii) How many such words contain the letter a?
   (iii) How many contain the letter a, b and c?
   (iv) How many begin with e and end with t?
   (v) How many begin with s and end with t? [(MATH2103.4)(Understand/LOCQ)]

   (b) Solve the recurrence relation:
   $a_n = 5\,a_{n-1} - 6a_{n-2} + 3n$, given that $a_0 = 0$ and $a_1 = 1$.
   [(MATH2103.4)(Evaluate/HOCQ)]

5 + 7 = 12

7. (a) How many integer solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 20$, where
   (i) $x_1, x_2 \geq -3, \quad x_3, x_4 \geq 0, \quad x_5 \geq 3$?
   (ii) $x_1 \geq 3, \ \ x_2 \geq 2, \ x_3 \geq 4, \ x_4 \ \geq 6$ and $x_5 \geq 0$? [(MATH2103.4)(Evaluate/HOCQ)]

   (b) A total of 1232 students have taken a course in Bengali, 879 have taken a course in English, and 114 have taken a course in Hindi. Further, 103 have taken courses in both Bengali and English, 23 have taken courses in both Bengali and Hindi, and 14 have taken courses in both English and Hindi. If 2092 students have taken at least one of Bengali, English and Hindi, how many students have taken a course in all three languages? [(MATH2103.4) (Analyze/IOCQ)]

(3 + 3) + 6 = 12

Group – E

8. (a) Prove that the premises $a \rightarrow (b \rightarrow c)$, $d \rightarrow (b \wedge \sim c)$ and $(a \wedge d)$ are inconsistent.
   [(MATH2103.5, MATH2103.6)(Create/HOCQ)]

   (b) Find the conjunctive normal form without using truth tables of the following statement:
   $$\big(p \wedge \sim(q \vee r)\big) \vee \Big(\big((p \wedge q) \vee \sim r\big) \vee p\Big)$$
   [(MATH2103.5, MATH2103.6)(Evaluate/HOCQ)]

6 + 6 = 12

9. (a) Construct truth table to find whether the conclusion c follows from the premises H_1 and H_2 where $H_1{:}\sim p$, $\ H_2{:}\, p \vee q$ and $c{:}\, p \wedge q$.
   [(MATH2103.5, MATH2103.6)(Analyze/IOCQ)]

   (b) Write down the converse, inverse and contra-positive of the given statement and also represent them symbolically.
   Statement: "If today is Independence Day, then tomorrow is Monday."
   [(MATH2103.5, MATH2103.6)(Remember/LOCQ)]

(c) Obtain the DNF (Disjunctive Normal Form) of the following statement:

$$p \rightarrow \{(p \rightarrow q) \wedge \sim (\sim q \vee \sim p)\}.$$

[(MATH2103.5, MATH2103.6)(Create/HOCQ)]

3 + 4 + 5 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	23.96	44.79	31.25

Course Outcome (CO):

After the completion of the course students will be able to
MATH2103. 1. Interpret the problems that can be formulated in terms of graphs and trees.
MATH2103. 2. Explain network phenomena by using the concepts of connectivity, independent sets, cliques, matching, graph coloring etc.
MATH2103. 3. Achieve the ability to think and reason abstract mathematical definitions and ideas relating to integers through concepts of well-ordering principle, division algorithm, greatest common divisors and congruence. MATH2103. 4. Apply counting techniques and the crucial concept of recurrence to comprehend the combinatorial aspects of algorithms.
MATH2103. 5. Analyze the logical fundamentals of basic computational concepts.
MATH2103. 6. Compare the notions of converse, contrapositive, inverse etc. in order to consolidate the comprehension of the logical subtleties involved in computational mathematics.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.