METHODS IN OPTIMIZATION
(MATH 4121)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The reduction ratio $\frac{I_0}{I_n}$ for Fibonacci search algorithm where I_0 is the initial interval of uncertainty and I_n is the final interval of uncertainty is

(a) F_n (b) 1 (c) F_{n-1} (d) $(F_n)^2$.

(ii) The quadratic form $Q(x, y, z) = x^2 + y^2 + 2xy + 2xz + 2yz$ *is*

(a) positive definite (b) positive semi definite
(c) negative definite (d) indefinite.

(iii) If all a_{ij} values in the incoming variable column of the simplex table are negative, then

(a) solution is unbounded (b) there are multiple solutions
(c) there exists no solution (d) the solution is degenerate

(iv) The degeneracy in the transportation problem indicates that

(a) dummy allocation needs to be added (b) solution is never optimal
(c) problem has two feasible solutions (d) multiple optimal solution exists.

(v) If $(-1, 1)$ is a stationary point of the function $f(x, y)$ such that

$$\frac{\partial^2 f}{\partial x^2} = x^2 + y^2, \frac{\partial^2 f}{\partial y^2} = x^2 \text{ and } \frac{\partial^2 f}{\partial x \partial y} = xy$$

then

(a) $(-1, 1)$ is a local maximum point but not global
(b) $(-1, 1)$ is a saddle point
(c) $(-1, 1)$ is a local minimum point but not global
(d) $(-1, 1)$ is a local minimum point.

(vi) The function $f(x) = 2x^3 - 3x^2$ is

(a) convex for all x (b) convex for all $x \geq \frac{1}{2}$
(c) concave for all $x \geq \frac{1}{2}$ (d) concave for all x.

(vii) In a fair game the value of the game is

(a) 1 (b) 0 (c) unbounded (d) 2.

(viii) The range of λ for which the following payoff matrix is strictly determinable is

PLAYER B (columns), PLAYER A (rows)

λ	6	2
-1	λ	-7
-2	4	λ

(a) $\lambda \geq -1$ (b) $\lambda \leq 2$ (c) $-1 \leq \lambda \leq 2$ (d) for any value of λ.

(ix) Given the optimization problem
Optimize $f(x,y,z)$ subject to $g_i(x,y,z) = b_i, i = 1,2;$
then the order of the bordered Hessian matrix of the Lagrangian function is
(a) 3×3 (b) 4×3 (c) 5×4 (d) 5×5.

(x) The Hessian matrix of the function $f(x,y)$ is given by
$H(f(x,y)) = \begin{pmatrix} -12x^2 - 2 & 2 \\ 2 & -2 \end{pmatrix}.$
If $(0,0)$ is a stationary point, then this point would be
(a) a local minimum but not global minimum point
(b) a global maximum point
(c) a saddle point
(d) a local maximum but not global maximum point.

Group – B

2. (a) A company makes two kinds of leather belts A and B. Their respective unit profits are $Rs. 4$ and $Rs. 3$. One belt of type A requires 2 hours and one belt of type B requires 1 hour of time in making. The total man-hours available are 1000 per day. Due to insufficient supply of leather, the company can make only 800 belts per day. Only 400 buckles for type A and 700 buckles of type B are available. Formulate the problem as an LPP and solve it graphically.
[(MATH4121.1, MATH4121.2)(Create/HOCQ)]

(b) Use simplex method to solve the following L.P.P.:
Maximize $z = 3x_1 + 2x_2 + 5x_3$
subject to the constraints

$$x_1 + 2x_2 + x_3 \leq 430$$
$$3x_1 + 2x_3 \leq 460$$
$$x_1 + 4x_2 \leq 420$$
$$x_1, x_2, x_3 \geq 0.$$

[(MATH4121.1, MATH4121.2)(Apply/IOCQ)]

5 + 7 = 12

3. (a) Use Big-M method to solve the following L.P.P.:
Maximize $z = 4x_1 + 5x_2 - 3x_3 + 50$
subject to the constraints

$$x_1 + x_2 + x_3 = 10$$
$$x_1 - x_2 \geq 1$$
$$2x_1 + 3x_2 + x_3 \leq 40$$
$$x_1, x_2, x_3 \geq 0.$$

[(MATH4121.1, MATH4121.2)(Apply/IOCQ)]

(b) Find the dual of the given LPP:
Maximize $z = 3x_1 + 4x_2 - x_3$
subject to the constraints

$$x_1 + 2x_2 + x_3 \leq 5$$
$$-8x_1 + 6x_2 + x_3 = 19$$
$$3x_1 + x_2 - x_3 \geq 7$$

$x_1, x_2 \geq 0$ and x_3 is unrestricted in sign.
[(MATH4121.1, MATH4121.2)(Understand/LOCQ)]

6 + 6 = 12

Group – C

4. (a) Find the optimal solution of the given transportation matrix:

	D_1	D_2	D_3	D_4	Availability
S_1	75	39	48	57	23
S_2	10	48	64	9	44
S_3	0	50	24	30	33
Demand	23	31	16	30	

[(MATH4121.1, MATH4121.2 MATH4121.3, MATH4121.4)(Evaluate/HOCQ)]

(b) The owner of a small machine shop has four mechanics available to assign jobs for the day. Five jobs are offered with expected profit for each machine in each job which is as follows:

Jobs

Mechanics

	A	B	C	D	E
1	62	78	50	101	82
2	71	84	61	73	59
3	87	92	111	71	81
4	48	64	87	77	80

By using the assignment method find the assignment of machines to the job that will result in a maximum profit.

[(MATH4121.1, MATH4121.2, MATH4121.3, MATH4121.4)(Evaluate/HOCQ)]

7 + 5 = 12

5. (a) Use graphical method to solve the following game and find the value of the game:

PLAYER B

PLAYER A

6	5
3	6
8	4
7	-1

[(MATH4121.1, MATH4121.2 MATH4121.3, MATH4121.4)(Understand/LOCQ)]

(b) Use dominance to reduce the following pay-off matrix to a 2×2 game and hence find the optimal strategies and the value of the game:

PLAYER B

PLAYER A

	B_1	B_2	B_3	B_4
A_1	5	-10	9	0
A_2	6	7	8	1
A_3	8	7	15	1
A_4	3	4	-1	4

[(MATH4121.1, MATH4121.2 MATH4121.3, MATH4121.4)(Apply/IOCQ)]

6 + 6 = 12

Group – D

6. (a) Verify whether the following function is convex or concave and find the maximum and minimum (if any)

$$f(x_1, x_2, x_3) = x_1 + 2x_3 + x_2x_3 - x_1^2 - x_2^2 - x_3^2.$$

[(MATH4121.6)(Remember/LOCQ)]

(b) Use Kuhn-Tucker conditions to solve the following non-linear programming problem:
Maximize $z = -x_1^2 - x_2^2 - x_3^2 + 4x_1 + 6x_2$
subject to the constraints

$$x_1 + x_2 \leq 2$$
$$2x_1 + 3x_2 \leq 12$$
$$x_1, x_2 \geq 0$$

[(MATH4121.6)(Evaluate/HOCQ)]

4 + 8 = 12

7. Use the method of Lagrange multipliers to solve the following non-linear programming problem. Does the solution maximize or minimize the objective function?
Optimize $z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$
subject to the constraint

$$x_1 + x_2 + x_3 = 15$$
$$2x_1 - x_2 + 2x_3 = 20$$
$$x_1, x_2, x_3 \geq 0.$$

[(MATH6121.6)(Evaluate/HOCQ)]

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Group – E

8. Use Fibonacci search algorithm to maximize $f(x) = x^4 - 14x^3 + 60x^2 - 70x$ in $[0,\ 2]$ using 6 functional evaluations. Consider $\in = 0.001$

[(MATH4121.1, MATH4121.5)(Apply/IOCQ)]

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9. Use Golden section search algorithm to minimize $f(x) = -\frac{1}{(x-1)^2}(\log x - 2\frac{x-1}{x+1})$ in the range $[1.5, 4.5]$ taking a tolerance of 1.00. [(MATH4121.1, MATH4121.5)(Apply/IOCQ)]

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Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	16.67	44.79	38.54

Course Outcome (CO):

After the completion of the course students will be able to
MATH4121.1 Describe the way of writing mathematical model for real-world optimization problems.
MATH4121.2 Identify Linear Programming Problems and their solution techniques.
MATH4121.3 Categorize Transportation and Assignment problems.
MATH4121.4 Apply the way in which Game Theoretic Models can be useful to a variety of real-world scenarios in economics and in other areas.
MATH4121.5 Apply various optimization methods for solving realistic engineering problems and compare their accuracy and efficiency.
MATH4121.6 Convert practical situations into non-linear programming problems and solve unconstrained and constrained programming problems using analytical techniques.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.