ENGINEERING COMPUTATIONAL TECHNIQUES
(MECH 4124)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and
any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The Euler's approximation method to solve the differential equation is
- (a) not used for first order differential equation
- (b) derived based on linear approximation
- (c) having the round off error only
- (d) used to get accurate solution.

(ii) The number of significant digits in 0.002345 is
(a) 3 (b) 5 (c) 4 (d) 6.

(iii) Which one is an iterative process to solve the linear equations?
(a) Gauss Elimination (b) LU Decomposition
(c) Gauss Seidel (d) Matrix inversion.

(iv) The solution is achieved from the tangent of the curve at a point. This is called
(a) Newton Raphson method (b) False Position Method
(c) Bi section Method (d) Graphical method.

(v) The motion of a particle $x(t)$ moving along the $x-$axis, whose velocity at time t is specified by the continuous function $f(t)$ and whose initial position is specified as $x(t = t_0) = x_0$, is described as
(a) $x(t) = \int_{t_0}^{t} f(\tau)d\tau$ (b) $x(t) = \int_{t}^{t_0} f(\tau)d\tau$
(c) $x(t) = x_0 + \int_{t_0}^{t} f(\tau)d\tau$ (d) $x_0 = x(t) + \int_{t_0}^{t} f(\tau)d\tau$

(vi) Lagrange interpolating polynomial is used to find
- (a) the root of an equation
- (b) the solution of two linear equations
- (c) the functional value at a point when a set of data is given
- (d) the integral value of a function.

(vii) In polynomial regression method, to fit a 3rd order polynomial no of equations required is
(a) 2 (b) 3 (c) 4 (d) 5.

(viii) In Simpson's 1/3rd rule to compute the integral value of a function,
- (a) the sum of odd point functional values are multiplied by 4
- (b) the sum of odd point functional values are multiplied by 2
- (c) the sum of even point functional values are multiplied by 4
- (d) the sum of even point functional values are multiplied by 3.

(ix) The order and degree of the ordinary differential equation $\left(\frac{d^3y}{dx^3}\right)^4 - 6x^2\left(\frac{dy}{dx}\right)^8 + e^y = \sin xy$ are respectively

(a) 4, 8 (b) 3, 8 (c) 3, 4 (d) 4, 3.

(x) The linear and homogeneous second-order PDE $A\frac{\partial^2\varphi}{\partial x^2} + B\frac{\partial^2\varphi}{\partial x\partial y} + C\frac{\partial^2\varphi}{\partial y^2} + D\frac{\partial\varphi}{\partial x} + E\frac{\partial\varphi}{\partial y} + F\varphi + G = 0$ is classified as hyperbolic in characteristics if

(a) $B^2 - 4AC > 0$ (b) $B^2 - 4AC < 0$
(c) $B^2 - 4AC = 0$ (d) $B^2 - 4AC \geq 0$.

Group – B

2. (a) Newton's law of cooling says that the temperature of a body changes at a rate proportional to the difference between its temperature and that of the surrounding medium:

$$\frac{dT}{dt} = k(T - T_a)$$

where, T = the temperature of the body (^{0}C), t = time (min), k is proportionality constant (per minute) and T_a = the ambient temperature (^{0}C). Suppose that a cup of coffee originally has a temperature of 85^{0}C. Use Euler's numerical method to compute the temperature from t = 0 to 10 min using a step size of 2 min if T_a = 25^{0}C and k = 0.015 /min. [(CO1)(Evaluate/HOCQ)]

(b) Explain total numerical error with diagram. [(CO1)(Understand/LOCQ)]

8 + 4 = 12

3. (a) Using the Newton-Raphson method determine the highest real root of $f(x) = x^3 - 6x^2 + 11x - 6.1$ with an initial assumption $x_0 = 3.5$. The solution must be correct with four decimal places. [(CO2)(Analyze/IOCQ)]

(b) Using LU decomposition method solve the following set of linear equations.

$$6x - 5y + 3z = -25.5$$
$$-7x + 2y + 6z = 41$$
$$3x + y - 9z = -28.5$$

[(CO2)(Analyze/IOCQ)]

5 + 7 = 12

Group – C

4. (a) Using polynomial regression, fit a parabola through certain points and find out the equation of the parabola for the following data. [(CO3)(Analyze/IOCQ)]

x	2	3.5	4	5.5	6	7.5
y	6.4	14.6	21.8	34.6	42.5	56.7

(b) Use linear regression to find out the equation of the straight line.

x	2.5	4	5.5	6	7.5	9
y	12	17	21	28	34	37

[(CO3)(Analyze/IOCQ)]

8 + 4 = 12

5. (a) Using the Newton's interpolating polynomial method, calculate f(4) from the following data: [(CO3)(Analyze/IOCQ)]

x	1	2	3	5	7	8
y	3	6	19	99	291	444

(b) The following data defines the sea level concentration of dissolved oxygen for fresh water as a function of temperature.

T (^{0}C)	0	8	16	24	32	40
C (mg/L)	14.62	11.84	9.87	8.42	7.31	6.41

Estimate C (29^{0}C) using Lagrange interpolating polynomial. [(CO3)(Solve/IOCQ)]

6 + 6 = 12

Group – D

6. (a) Using Simpson's 1/3rd rule, evaluate the following integral. Take n = 4:
$\int_{-2}^{6}(1+\cos x)\,dx$ [(CO4)(Solve/IOCQ)]

(b) Evaluate the integral value of following function using 2 point Gauss Quadrature:
$f(x) = \frac{e^x \sin x}{1+x^2}$ [(CO4)(Evaluate/HOCQ)]

5 + 7 = 12

7. (a) Using the Taylor series approximation method evaluate the integral of the following IVP:

$$y' = 2x^2 + 3y^2;\ y(0) = 0.$$

Find the value of $y(x = 1)$. [Determine the first three non-zero terms].

[CO5/Solve/IOCQ]

(b) Use the classical 4th order Runge-Kutta method to find the solution of the following IVP on $[0, 0.4]$:

$$y' = 0.4y^2 + x^2;\ y(0) = -1.$$

Show the calculations clearly in tabular form till the 4th iteration step. Take $h = 0.1$.

[(CO3)(Understand/LOCQ)]

4 + 8 = 12

Group – E

8. (a) Evaluate the integral of the IVP $y' = 2x + y;\ y(0) = 2;\ h = 0.1$ by the modified Euler's method at x = 0 (0.1) 0.3. [CO5/Solve/IOCQ]

(b) Solve the initial boundary-value problem $\frac{\partial u}{\partial t} = \frac{\partial^2 f}{\partial x^2}$; $0 \le x \le 1, t > 0$ by the Schmidt explicit method at $t = 1/10\,, 2/10\,, 3/10$.
The two boundary conditions are $u(x = 0, t) = 0$ and $u(x = 1, t) = 0$. The initial condition is $u(x, t = 0) = \sin \pi x\,;\ 0 \le x \le 1$. Consider the spatial increment $h = 1/3$ and the time increment $k = 1/10$. [CO6/Apply/IOCQ]

4 + 8 = 12

9. (a) What are interpolating functions or shape functions? Are the shape functions continuous over the whole domain? [CO6/Formulation/HOCQ]

(b) Solve the following equation using the Galerkin weighted residual method by using a two-parameter solution:

$$\frac{dy}{dx} = 45(1 + \cos x) - 0.04y; 0 \le x \le 2\pi; y(0) = 150$$

[CO6/Formulate/HOCQ]

(2 + 1) + 9 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	12.5	59.37	28.13

Course Outcome (CO):

After the completion of the course students will be able to

CO 1: Apply mathematical models for numerical solutions and classify different types of error.

CO 2: Solve a system of linear algebraic equations by different methods and find out the roots.

CO 3: Implement the regression and interpolation methods for curve fitting and solve different types of optimization problems.

CO 4: Use different numerical integration methods for practical problems.

CO 5: Classify Initial and Boundary value problems to select appropriate solution strategies, and solve Eigenvalue problems applied to physical systems.

CO 6: Apply the Finite Difference Method and the Finite Element Method to formulate and develop solutions for one-dimensional and two-dimensional problems in partial differential equations.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.