FORMAL LANGUAGE & AUTOMATA THEORY
(CSEN 3102)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

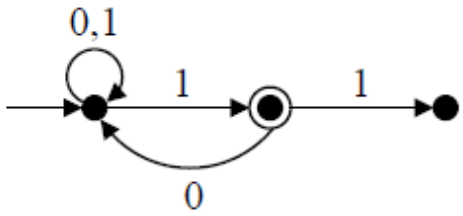
Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) What is the language of the following NFA over the alphabet {0, 1}?



(a) The set of all strings not containing the pattern 11
(b) The set of all strings ending with 1
(c) The set of all strings ending with 01
(d) The set of all strings not containing the pattern $(01)^*1$.

(ii) Which of the following statements is true for a CFG G with start symbol S and with the only productions $S \rightarrow aS \mid bS \mid a$?
(a) L (G) is a CFL but not regular
(b) L (G) is regular but not a CFL
(c) L (G) is a CFL and regular
(d) L (G) is neither a CFL nor regular

(iii) Which of the following is the correct regular expression of the following language $L = \{a^n b^m \mid n>=4, m<=3\}$?
(a) aaaa*(λ+b+bb+bbb)
(b) aaaa*+b+bb+bbb
(c) aaaaa*(λ+b+bb+bbb)
(d) None of these.

(iv) If a Context Free Grammar is in Chomsky Normal Form then a production
(a) may contain a single non-terminal in the RHS
(b) may contain both terminal and non-terminal symbols in the RHS
(c) may be of the form $A \rightarrow \varepsilon$ where A is not the start symbol
(d) none of these.

(v) Consider the CFG G = ({S}, {a, b}, P, S), with following productions:
$S \rightarrow aSa \mid bSb \mid \lambda$
Which of the following is the correct representation of the language of G?
(a) $L(G) = \{w \in \{a, b\}^* \mid n_a(w) = n_b(w)$ and $n_a(v) >= n_b(v)$, where v is any prefix of w
(b) $L(G) = \{ww^R \mid w \in \{a, b\}^*\}$
(c) $L(G) = \{a^{2n}b^m \mid n>= 0, m>= 0\}$
(d) None of these.

(vi) If the original nondeterministic finite state automata (ndfsa) has n states, the corresponding deterministic finite state automata (dfsa) cannot have
(a) more than n states (b) more than 2n states
(c) more than 2^n states (d) more than (n-1) states

(vii) If a Context Free Language L_3 has a Type 2 grammar G which is ambiguous then there must be a terminal string α such that
(a) α has two different derivation trees
(b) α has two different rightmost derivations
(c) α has two different leftmost derivations
(d) All of the above.

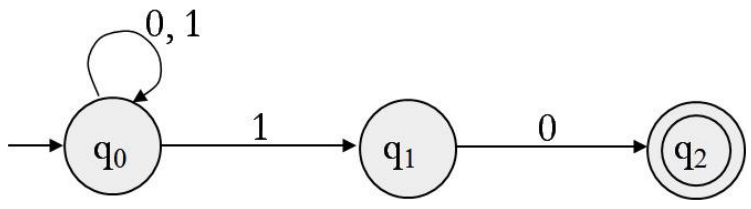
(viii) Consider the following identities for regular expressions:
(i) $(r + s)^* = (s + r)^*$ (ii) $(r^*)^* = r^*$ (iii) $(r^* s^*)^* = (r + s)^*$
Which of the above identities are true?
(a) (i) and (ii) only (b) (ii) and (iii) only
(c) (iii) and (i) only (d) (i), (ii) and (iii).

(ix) Which one of the following languages over the alphabet {0, 1} is described by the regular expression: (0+1)*0(0+1)*0(0+1)*?
(a) The set of all strings containing the substring 00
(b) The set of all strings containing at most two 0's
(c) The set of all strings containing at least two 0's
(d) The set of all strings that begin and end with either 0 or 1.

(x) Which of the following pairs of machines given below *do not* have equal computing power?
(a) Deterministic and Nondeterministic finite state automata
(b) Deterministic and Nondeterministic pushdown automata
(c) Deterministic and Nondeterministic Turing Machine
(d) Multi tape Turing machine and Universal Turing machine.

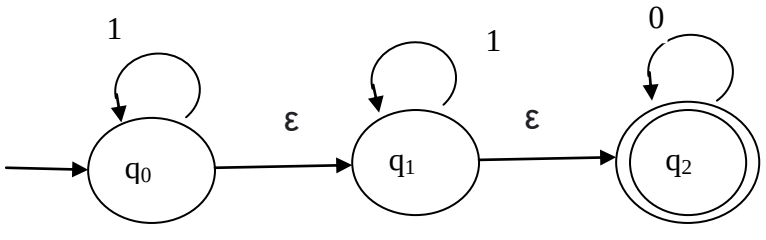
Group- B

2. (a) The state transition diagram of a non-deterministic finite state acceptor (nfa) M_2 is given below. Here q_0 is the start state, and q_2 is the only goal state. First, derive the regular expression of the language accepted by this nfa. Then convert M_2 to an equivalent deterministic finite state acceptor (dfa) M_3.



[(CSEN3102.2)(Apply/IOCQ)]

(b) Remove ε-transitions from the given NFA



[(CSEN3102.2)(Apply/IOCQ)]
(2 + 4) + 6 = 12

3. (a) What do you mean by a non-deterministic finite automata (NFA)? When is a string accepted by a given NFA? [(CSEN3102.1)(Remember, Understand/LOCQ)]

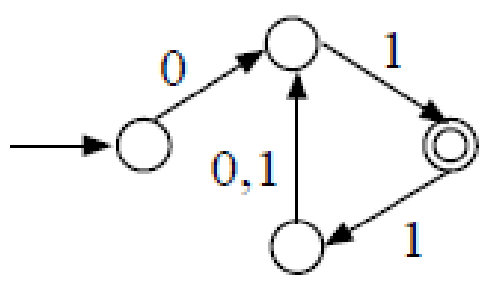
(b) Consider Σ = {0, 1}. Design a DFA accepting the set of all strings, when interpreted in reverse as a binary integer, is divisible by 3. You may give the state transition diagram or state transition table. [(CSEN3102.6)(Apply/IOCQ)]

(c) What do you mean by the complement of the language accepted by any NFA? [(CSEN3102.3)(Understand/IOCQ)]

(2 + 2) + 6 + 2 = 12

Group – C

4. (a) Consider the following NFA on Σ = {0, 1}:



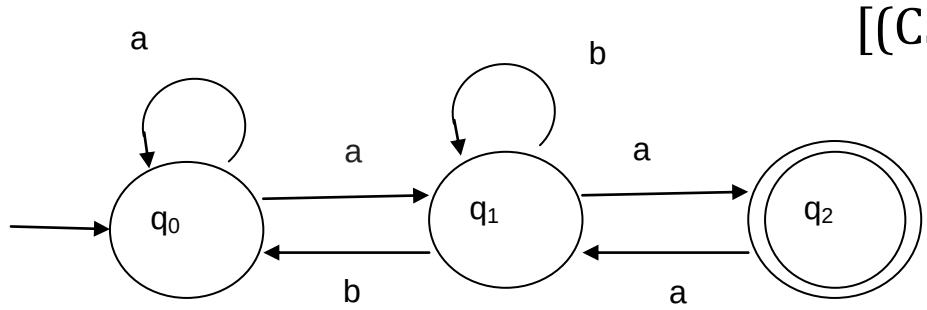
Find the regular expression that represents the same language that the above NFA represents. Show the steps in detail. [(CSEN3102.3)(Apply/IOCQ)]

(b) Using the pumping lemma, prove that the language L2 = {$a^i b^j$ | i, j >= 0, and |i − j| is a prime} is not regular. (Note that 1 is not treated as a prime.) [(CSEN3102.5)(Evaluate/HOCQ)]

6 + 6 = 12

5. (a) Prove, (1 + 00*1) + (1 + 00*1)(0 + 10*1)* (0 + 10*1) =0*1(0 + 10*1)*. [(CSEN3102.3)(Apply/IOCQ)]

(b) Consider the transition system given in the Figure below.
Prove that the strings recognized are (a + a(b + aa)*b)* a(b + aa)* a. [(CSEN3102.6)(Apply/IOCQ)]



(c) What do you mean by right linear grammar? Give an example. [(CSEN3102.1) (Understand/LOCQ)]

3 + 6 + 3 = 12

Group - D

6. (a) When a Context free grammar will be called ambiguous? Using an example explain your answer. [(CSEN3102.1)(Understand/LOCQ)]

(b) Simplify the given Context free grammar by removing all unit productions

S → Aa | B
B → A | bb
A → a| bc | B. [(CSEN3102.6)(Apply/IOCQ)]

(c) Consider the following context-free grammar *G*:

S →AB

$A \rightarrow aA \mid bB \mid b$
$B \rightarrow b$

Convert *G* to Greibach Normal Form (GNF). [(CSEN3102.5)(Evaluate/HOCQ)]

(2 + 2) + 4 + 4 = 12

7. (a) Consider the following context free language (CFL):
L3 = $\{a^n b^n c^m\} \cup \{a^n b^m c^m\}$, where n, m>=0
Find a context free grammar (CFG) G for L3. [(CSEN3102.4)(Apply/IOCQ)]
(b) Show that the grammar you have obtained in part (a) is ambiguous. [(CSEN3102.3)(Apply/IOCQ)]
(c) What do you mean by inherently ambiguous context free language? [(CSEN3102.1)(Remember/LOCQ)]

6 + 4 + 2 = 12

Group – E

8. (a) Design a Turing machine M_5 which can accept the language
$L_{15} = \{a^n b^n : n>=1\}$. [(CSEN3102.6)(Create/HOCQ)]
(b) Now show that M_5 accepts "aaabbb" but rejects "abab" and "aab". [(CSEN3102.5)(Analyze/IOCQ)]

7 + 5 = 12

9. (a) Design a Turing Machine for the following language L5:
L5 = $\{w\$w \mid w \in \Sigma^*\}$, where $\Sigma = \{a, b, \$\}$. [(CSEN3102.6)(Apply/IOCQ)]
(b) Distinguish between Turing Recognizable and Turing Decidable languages. [(CSEN3102.5)(Understand/LOCQ)]

8 + 4 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	17.71	64.58	17.71

Course Outcome (CO):

After completion of the course, students will be able to:

CSEN3102.1. Recall the basic characteristics of various types of machines, languages and grammars.
CSEN3102.2. Compare different computational models, languages and grammars based on their properties and behaviors.
CSEN3102.3. Apply formal mathematical methods to prove properties of languages, grammars, and automata.
CSEN3102.4. Apply the knowledge of theory of computation to an engineering application (e.g. designing the compilers).
CSEN3102.5. Classify formal languages and Evaluate whether a language/grammar belongs to a given type or not.
CSEN3102.6. Design automata for given languages/grammars. Generate languages/grammars for a given automaton and Construct grammars for languages and vice versa.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.