PHYSICS - I
(PHYS 1001)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and <u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) If $\vec{\nabla}.\vec{A} = \rho$ (constant), for a closed surface S enclosing a volume V, then
(a) $\oint \overrightarrow{A}\, dV = \vec{A}\, S$ (b) $\oint \overrightarrow{A}\, dV = 0$
(c) $\oint \overrightarrow{A}.d\vec{S} = \rho V$ (d) $\oint \overrightarrow{A}\, dV = \vec{A}\, V$

(ii) The relation between a magnetic field $\vec{B}$ and corresponding vector potential $\vec{A}$ can be given by
(a) $\int \vec{A} \times \overrightarrow{ds} = \int \vec{B} \times \overrightarrow{dl}$ (b) $\int \vec{A}.\overrightarrow{ds} = \int \vec{B}.\overrightarrow{dl}$
(c) $\int \vec{A}.\overrightarrow{dl} = \int \vec{B}.\overrightarrow{ds}$ (d) $\int \vec{A} \times \overrightarrow{dl} = \int \vec{B} \times \overrightarrow{ds}$

(iii) Poisson equation in electrostatic is given by the following relation
(a) $\nabla^2\varphi = 0$ (b) $\nabla^2\varphi = \frac{-\rho}{\epsilon_0}$ (c) $\vec{\nabla}.\vec{E} = \frac{-\rho}{\epsilon_0}$ (d) $\vec{\nabla}.\vec{E} = \epsilon_0\rho$

(iv) A uniform magnetic field is acting along the positive Z-axis. The value of vector potential ($\vec{A}$) at a distance r from the Z-axis has the magnitude
(a) 2Br (b) Br (c) B/2r (d) Br/2

(v) For which of the following forces, angular momentum will be conserved
(a) $\vec{F} = 3cos\theta\hat{r}$ (b) $\vec{F} = \frac{5}{r}\hat{\theta}$ (c) $\vec{F} = \frac{-sin\theta}{r}\hat{r}$ (d) $\vec{F} = logr\,\hat{r}$

(vi) Damping constant γ resembles the dimension of
(a) length (b) mass (c) time (d) natural frequency

(vii) If for any vector field $\vec{F}, \vec{\nabla} \times \vec{F} = \alpha\vec{F}$, the vector field is
(a) source (b) sink (c) solenoidal (d) irrotational.

(viii) The gradient of a scalar field is
(a) scalar (b) vector (c) zero always (d) none of these.

(ix) A time varying magnetic field $\vec{B} = B_0 \cos(2z - \omega t)\,\hat{\imath}$ is producing an electric field $\vec{E} = E_0 \cos(2z - \omega t)\,\hat{\jmath}$. Then the magnitude of ω is
(a) $\frac{E_0}{B_0}$ (b) $2\,\frac{E_0}{B_0}$ (c) $\frac{B_0}{E_0}$ (d) $2\,\frac{B_0}{E_0}$

(x) An electrostatics field is always
(a) a source field (b) a sink field
(c) an irrotational field (d) a solenoidal.

Group – B

2. (a) Show that $\vec{\nabla}\varphi$ is a vector perpendicular to the surface $\varphi(x, y, z) = constant$. [(CO1)(Remember/LOCQ)]
(b) Find the directional derivative of $\varphi = x^2yz + 4xz^2 - 6$ at $(1, -2, -1)$ in the direction of $2\hat{\imath} - \hat{\jmath} - 2\hat{k}$. [(CO1)(Remember/LOCQ)]
(c) For the vector $\vec{V} = 2yz\hat{\imath} + 3xz\hat{\jmath} + 4xy\hat{k}$, simplify the value of $\vec{\nabla}.\left(\vec{\nabla} \times \vec{V}\right)$. [(CO1)(Analyse/IOCQ)]
(d) Construct the expression of del operator $(\vec{\nabla})$ for plane polar coordinates system. Find the value of $\vec{\nabla}.\vec{r}$, using plane polar coordinates where $\vec{r}$ is the position vector. [(CO1)(Create/HOCQ)]

2 + 3 + 3 + (2 + 2) = 12

3. (a) The position vector $\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath} + z(t)\hat{k}$ of a particle moving relative to Earth surface is given by the following three equations: $\ddot{x} = 2\omega\dot{y}\cos\lambda$, $\ddot{y} = -2\omega\dot{z}\sin\lambda - 2\omega\dot{x}\cos\lambda$ and $\ddot{z} = -g + 2\omega\dot{y}\sin\lambda$. Here, ω is the Earth's angular velocity with respect to the Z axis of the fixed frame, λ is the co-latitude of the initial position of the particle and g is acceleration due to gravity when earth is not rotating. If the particle is dropped from rest from a height h then conclude that it will be deflected towards eastward direction in the northern hemisphere. [(CO2)(Evaluate/HOCQ)]
(b) From the fundamental definition of areal vector defend Kepler's second law. [(CO2)(Analyse/IOCQ)]
(c) (i) Write down the differential equation of the orbit of a planet under the influence of a central force explaining all terms.
(ii) Using the above equation justify that in absence of a central force the planet will move in a straight line. [(CO2)(Evaluate/HOCQ)]

4 + 4 + (1 + 3) = 12

Group – C

4. (a) A series LCR circuit is subjected to a DC voltage V.
(i) Construct a differential equation for the charge stored in the capacitor at any time t.
(ii) Estimate the condition for which the charge at any time is oscillatory.
(iii) If the value of L, C, R are α H, β F and γ Ω respectively then estimate the frequency and Q-factor of the circuit. [(CO4)(Analyse/IOCQ)]
(b) The displacement of a particle of mass 2 gm is given by
$x(t) = 5\, e^{-3t} \cos\left(\pi t - \frac{\pi}{3}\right)$, estimate
(i) the damping constant,
(ii) the natural frequency,
(iii) the logarithmic decrement and the amplitude relaxation time of the system.
[(CO4)(Create/HOCQ)]

(c) Demonstrate with a plot the velocity amplitude versus the frequency of the external periodic force of a forced harmonic oscillator for two different values of the damping constant. [(C04)(Understand/LOCQ)]

(2 + 2 + 3) + (1 + 1 + 2) + 1 = 12

5. (a) (i) A linearly polarized light wave of angular frequency ω is propagating along the direction of the vector $\vec{n} = 2\hat{\imath} + 2\hat{\jmath} + \hat{k}$. The light vector is directed along the vector $\vec{e} = \hat{\imath} - \hat{\jmath}$. Construct the expression of the light vector with amplitude E_0 and wavelength λ.

(ii) Find the plane of vibration of this light vector. [(CO6)(Apply/IOCQ)]

(b) Evaluate the ratio of spontaneous emission rate to the stimulated emission rate at room temperature of 30°C, for visible light of frequency 10^{14}Hz and microwave of frequency 10^9 Hz. Comment on the result. [(CO6)(Evaluate/HOCQ)]

(c) A step index optical fibre has a core of refractive index 1.45 and a cladding of refractive index 1.35. If the fibre is used in a water environment, evaluate the numerical aperture and the acceptance angle of the fibre. Refractive index of water is 1.33.

[(CO6)(Evaluate/HOCQ)]

(3 + 2) + (3 + 1) + 3 = 12

Group – D

6. (a) Define electrostatic field. Establish the Poisson's equation starting from integral form of Gauss's law of electrostatics. Under what condition poisson's equation does reduce to Laplace's equation? [(CO4)(Remember/LOCQ)]

(b) Four point charges of magnitude -5C, 3C, 2C and -4C are located at four corners of a square of side 8m. Find the electric field at the center of the square (using vector method). [(CO4)(Remember/LOCQ)]

(c) A ring of radius R and uniform linear charge density λ is placed on YZ plane with the center coinciding with the origin. Evaluate the electrostatic field at $(x_0, 0, 0)$.

[(CO4)(Evaluate/HOCQ)]

(d) Show that potential due to a dipole placed at the origin at a large distance is given by $\varphi(r,\theta) = \frac{\vec{P}.\vec{r}}{4\pi\varepsilon_0 r^3}$, where $\vec{P}$ is the dipole moment and (r,θ) is the coordinates of the point where the potential has to be found out. [(CO4)(Remember/LOCQ)]

(1 + 2 + 1) + 2 + 3 + 3 = 12

7. (a) Consider a spherical charge distribution having radius R and charge density $\rho(r) = \rho_0 \left(1 + \frac{r^2}{R^2}\right)$. Find the electric field at any external point.

[(CO4)(Remember/LOCQ)]

(b) Two concentric sphere of radii R and nR (n≠1) are kept at potential V and $\frac{V}{n}$ respectively. Show that potential at any internal point r $(R < r < nR)$ is independent of n. and also determine the electric field at an internal point. [(CO4)(Evaluate/HOCQ)]

(c) A dielectric sphere of radius R, centered at the origin carries a polarization $\vec{P} = k\vec{R}$. Develop the total volume bound charge and the total surface bound charge.

[(CO5)(Apply/IOCQ)]

3 + (3 + 2) + (2 + 2) = 12

Group – E

8. (a) Define magnetostatic field induction. Show that two vector potentials $\overrightarrow{A_1} = k(-y\hat{\imath} + z\hat{\jmath})$ and $\overrightarrow{A_2} = k[(x-y)\hat{\imath} + (y+z)\hat{\jmath} + z\hat{k}]$ represent the same magnetic field. Explain the reason. [(CO4)(Remember/LOCQ)]
   (b) An infinitely long current carrying straight wire is kept along the Z axis. Find the magnetic induction vector at a point $(4, \frac{\pi}{6})$. [(CO4)(Understand/LOCQ)]
   (c) For linear isotropic material, show that $\mu = \mu_o(1+\chi)$, where the symbols have their usual meaning. [(CO5)(Analyse/IOCQ)]
   (d) The magnetic moment of a loop carrying a current I is given by the formula $\vec{m} = \frac{I}{2}\oint \overrightarrow{r'} \times \overrightarrow{dr'}$ where the symbols have their usual meanings. Find the magnetic moment for the circular loop of radius R carrying a current I along the clockwise direction. [(CO5)(Analyse/IOCQ)]

(1 + 3) + 3 + 3 + 2 = 12

9. (a) Explain the fact that a steady current represents a magnetostatics field. Show that the vector field given by $\vec{B} = \frac{\mu_0 \alpha}{4\pi}(-x\hat{\imath} + y\hat{k})$ represents a magnetostatics field. Find the corresponding current density. [(CO4)(Remember/LOCQ)]
   (b) A current carrying circular loop with the centre at the origin on XY-plane has the magnetic moment $\vec{m} = m\hat{k}$. The vector potential $\vec{A}$ due to this loop at a distance $\vec{r}$ is given by $\frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3}$. Find the magnetic field at a point P with position vector $\vec{r} = 3\hat{\imath} + 2\hat{\jmath}$. [(CO4)(Understand/LOCQ)]
   (c) A time varying magnetic field $\vec{B} = B_0 \sin\omega t\,(\hat{\imath} + \hat{\jmath})$ is allowed to pass through a square loop of area $a^2(\hat{\imath} + \hat{\jmath})$. Find the maximum value of induced emf in the loop. [(CO4)(Analyse/IOCQ)]

(3 + 2 +1) + 3 + 3 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	35.4	32.3	32.3

Course Outcome (CO):

After completing this course, the students will be able to:
1. Understand and apply Vector Calculus as tool for solving different physical problems.
2. Analyze the nature of central forces and rotating frame phenomenon to understand basic space science and real world applications understand basic space science and real world applications.
3. Interpret the different types of oscillatory motion and resonance.
4. Apply fundamental theories and technical aspect in the field of electricity and magnetism in solving real world problems in that domain magnetism in solving real world problems in that domain.
5. Understand the Electrical and Magnetic properties of different types of materials for scientific and technological use materials for scientific and technological use.
6. Develop Analytical & Logical skill in handling problems in technology related domain.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question