(vi) The solution of $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$ is

(a) z=f $f_1(y+x) + f_1(y-x)$, (b) z= $f_1(y+x) + f_2(y-x)$,
(c) z= $f_2(y+x) + f_2(y-x)$, (d) z= $f(x^2 - y^2)$.

(vii) P.I. of $(D^2 - D'^2)z = \cos(x+y)$ is

(a) $\frac{x}{2}\cos(x+y)$ (b) $\frac{x}{2}\sin(x+y)$
(c) $x\sin(x+y)$ (d) $x\cos(x+y)$

(viii) If $f(z) = \frac{1}{z^4 - 2z^3}$, then $z = 0$ is a pole of order

(a) 3 (b) 2 (c) 1 (d) 4.

(ix) If $F\{f(t)\}=F(s)$ and $a \neq 0$, then $F\{f(at)\}$ is

(a) $\frac{1}{a}F(s/a)$, (b) $-\frac{1}{a}F(s/a)$,
(c) $\frac{1}{|a|}F(s/a)$, (d) none of these.

(x) Residue of $f(z) = \frac{2 + 3\sin \pi z}{z(z-1)^2}$ at z=0 is

(a) 1, (b) 3, (c) 2, (d) i.

Group – B

2. (a) Show by definition that the function $f(z) = \begin{cases} \frac{z(z-\bar{z})}{2i|z|}, & \text{when } z \neq 0 \\ 0, & \text{when } z = 0 \end{cases}$

is continuous at z=0.

(b) Find the analytic function whose imaginary part is $\frac{x-y}{x^2+y^2}$.

6 + 6 = 12

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3. (a) Use Cauchy's Integral formula to evaluate $\int_C \frac{e^{2z}}{(z+1)^4} dz$, where C is the circle $|z| = 3$.

(b) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent's series valid for the region $1 < |z| < 3$.

(c) Find $\int_C \frac{z^2 + 2z + 1}{(z-1)z^2(z-2)} dz$, where C is the circle $|z| = 6$.

3 + 3 + 6 = 12

Group – C

4. (a) Find the Fourier series for the function

$$f(x) = \begin{cases} \pi + 2x, & -\pi < x < 0 \\ \pi - 2x, & 0 \leq x \leq \pi \end{cases}$$

Hence deduce that, $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\infty = \frac{\pi^2}{8}$.

(b) Find the Fourier sine transform of $f(x) = \frac{1}{x}$.

7 + 5 = 12

5. (a) Find the Fourier transform of the function $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$ and hence find $\int_{-\infty}^{\infty} \frac{\sin sa \cos sx}{s} ds$.

(b) Find the Fourier inverse transform of the function $F(s) = \frac{1}{s^2 + 4s + 13}$.

7 + 5 = 12

Group - D

6. (a) Prove that $\int_{-1}^{1} P_m(x)P_n(x)dx = 0$ if $m \neq n$ where $P_m(x)$ and $P_n(x)$ are Legendre polynomials.

   (b) Show that, $J_n(-x) = (-1)^n J_n(x)$.

7 + 5 = 12

7. (a) Find power series solution of the differential equation $(x^2+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - xy = 0$ about the point $x = 0$.

   (b) Express the polynomial $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre's Polynomials.

7 + 5 = 12

Group - E

8. (a) Form the partial differential equation by eliminating the arbitrary function $f(xy + z^2, x + y + z) = 0$.

   (b) Solve the following partial differential equation by Charpit's method $(p^2 + q^2)y = qz$

6 + 6 = 12

9. (a) Find the general solution of the following Partial Differential Equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y \sin x$.

   (b) Find the solution of $\frac{\partial^2 u}{\partial x^2} = h^2 \frac{\partial u}{\partial t}$ for which $u(0,t) = u(l,t) = 0, u(x,0) = \sin\frac{\pi x}{l}$ by method of separation of variables.

5 + 7 = 12

2016

MATHEMATICAL METHODS
(MATH 2001)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for the following: **10 × 1=10**

   (i) For what value of m, $3y - 5x^2 + my^2$ will be a harmonic function?
   (a) 5 (b) -5 (c) 0 (d) 3.

   (ii) If *C* be a closed curve traversed in the anticlockwise sense and enclosing the point $z = a$, and *f*(*z*) be analytic within and on *C*, then $\int_C \frac{f(z)}{(z-a)^3}dz =$
   (a) $\pi i f''(a)$ (b) $-\pi i f''(a)$
   (c) $f''(a)$ (d) none of these.

   (iii) The solution of $xp + yq = z$ is
   (a) $f(x, y) = 0,$ (b) $f\left(x/y, y/z\right) = 0,$
   (c) $f(xy, yz) = 0,$ (d) $f(x^2, y^2) = 0.$

   (iv) If *F*(*s*) be the Fourier transform of *f* (*t*), then the Fourier transform of $f'(t)$ is
   (a) *F*(*s*), (b) –*isF*(*s*), (c) *isF*(*s*) (d) none of these.

   (v) The regular singular point of the ODE $xy'' + 2y' - xy = 0$ is
   (a) $x = 1$ (b) $x = 2$ (c) $x = 0$ (d) none of these.