MATHEMATICAL & STATISTICAL METHODS
(MATH 2101)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) If a random variable X has Poisson distribution such that $P(1) = P(2)$, then $P(0) =$
(a) $\frac{1}{e}$ (b) $\frac{1}{e^2}$ (c) $\frac{1}{e^3}$ (d) e.

(ii) Which one of the following numerical techniques can be used for non-equispaced arguments?
(a) Newton's Forward Interpolation (b) Newton's Backward Interpolation
(c) Lagrange's Interpolation (d) Stirling's Interpolation.

(iii) If $f(x) = x^2$, then the second order divided difference for the points x_0, x_1, x_2 will be
(a) -1 (b) $\frac{-1}{x_1 - x_0}$ (c) 1 (d) $\frac{1}{x_2 - x_1}$.

(iv) If X is normally distributed with zero mean and unit variance, then the expectation of X^2 is
(a) 1 (b) 2 (c) 8 (d) 0.

(v) A complete integral of $z = px + qy + \sqrt{pq}$ is
(a) $z = ax + by + ab$ (b) $z = ax - by + ab$
(c) $z = ax + by + \sqrt{ab}$ (d) $z = ax^2 + by^2 + \sqrt{ab}$
where a and b are arbitrary constants and $p \equiv \frac{\partial z}{\partial x}$, $q \equiv \frac{\partial z}{\partial y}$.

(vi) $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = e^{x-y}$ has complementary function
(a) $z = \phi_1(y - x) + x\phi_2(y + x)$ (b) $z = \phi_1(y - x) + \phi_2(y + x)$
(c) $z = \phi_1(y - x) + x\phi_2(y - x)$ (d) $z = \phi_1(y - 2x) + \phi_2(y + x)$.

(vii) If $f(x) = x^4$ in $-1 \le x \le 1$, then the Fourier coefficent b_n is:
(a) 0 (b) 1 (c) 2 (d) 4.

(viii) For Fourier expansion of an odd function in $(-\pi, \pi)$
(a) a_0 is zero (b) a_n is zero
(c) b_n is zero (d) both a_0 and a_n are zero.

(ix) Find the correct median for the sample: 4, 5, 6, 6, 7, 9, 10, 13, 15, 22
(a) 9 (b) 8th term (c) 7 (d) 8.

(x) The function $f(x) = \begin{cases} -1, -2 < x < 0 \\ 1, 0 < x < 2 \end{cases}$ will generate
(a) triangular waveform (b) saw-tooth waveform
(c) square waveform (d) both (a) and (b).

Group – B

2. (a) Solve the following partial differential equation by applying Lagrange's method: $p + 3q = 5z + \tan(y - 3x)$ where $p \equiv \frac{\partial z}{\partial x}, q \equiv \frac{\partial z}{\partial y}$. [(MATH2101.5)(Apply/IOCQ)]
(b) Form a partial differential equation by eliminating arbitrary functions f and φ from $z = f(x + iy) + \varphi(x - iy)$ where $i^2 = -1$. [(MATH2101.5)(Create/HOCQ)]

6 + 6 = 12

3. (a) Find a complete integral of $q = 3p^2$ using appropriate method, where $p \equiv \frac{\partial z}{\partial x}, q \equiv \frac{\partial z}{\partial y}$. [(MATH2101.5)(Apply/IOCQ)]
(b) Solve: $(\mathrm{D}^2 - 6\mathrm{D}\mathrm{D}' + 9\mathrm{D}'^2)\mathrm{z} = 12\mathrm{x}^2$, where $\mathrm{D} \equiv \frac{\partial}{\partial \mathrm{x}}$, $\mathrm{D}' \equiv \frac{\partial}{\partial \mathrm{y}}$.
[(MATH2101.5)(Evaluate/HOCQ)]

6 + 6 = 12

Group – C

4. (a) Using Newton's divided difference formula, calculate the value of y when $x = 3$ for the following table:

x	:	0	1	2	4	5	6
y	:	1	14	15	5	6	19

[(MATH2101.1, MATH2101.2)(Remember /LOCQ)]
(b) Apply Simpson's one-third rule to evaluate $\int_0^6 \frac{dx}{(1+x)^2}$, taking six equal sub-intervals of [0,6], correct to three decimal places.
[(MATH2101.1, MATH2101.2)(Apply/IOCQ)]

6 + 6 = 12

5. (a) Applying Lagrange's interpolation method, calculate $f(5.5)$ for the following set of data:

x:	0	2	3	5	7
y:	1	47	97	251	477.

[(MATH2101.1, MATH2101.2)(Apply/IOCQ)]
(b) Compute the value of π from the relation $\frac{\pi}{4} = \int_0^1 \frac{\mathrm{dx}}{1+\mathrm{x}^2}$ using Trapezoidal rule with 10 sub-intervals. [(MATH2101.1, MATH2101.2)(Remember/LOCQ)]

6 + 6 = 12

Group – D

6. (a) If $f(x) = \begin{cases} \pi x, & 0 < x < 1 \\ \pi(2-x), & 1 < x < 2 \end{cases}$, then find the half-range cosine series and hence show by using the Parseval's Identity

$$\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots.$$
[(MATH2101.4)(Apply/IOCQ)]

(b) Find Fourier series of $f(x) = \begin{cases} 0, & \text{for } -2 < x < 0 \\ 1, & \text{for } 0 < x < 2 \end{cases}$ in $(-2, 2)$.
[(MATH2101.4)(Remember/LOCQ)]

6 + 6 = 12

7. (a) Derive the Fourier series of $f(x) = x + x^2$ in $(-\pi, \pi)$ of periodicity 2π and hence deduce $\sum \frac{1}{n^2} = \frac{\pi^2}{6}$. [(MATH2101.4)(Understand/LOCQ)]

(b) Find Fourier cosine series of $f(x) = e^x$ in $0 < x < \pi$.
[(MATH2101.4)(Remember/LOCQ)]

7 + 5 = 12

Group – E

8. (a) If X has exponential distribution with parameters a and b, then find
(i) $E(X)$ (ii) $Var\ (X)$. [(MATH2101.3, MATH2101.6)(Remember/LOCQ)]

(b) A fair coin is tossed 500 times. Using normal approximation to Binomial distribution find the probability of obtaining
(i) exactly 240 heads.
(ii) between 220 and 280 heads, both inclusive.
[(MATH2101.3, MATH2101.6)(Remember/LOCQ)]

6 + 6 = 12

9. (a) The expenditure of 1000 families is given below:

Expenditure (Rs.)	40 – 59	60 – 79	80 – 99	100 – 119	120 – 139
Frequencies	50	?	500	?	50

If the mean and the median of the distribution are both 87.5, then calculate the missing frequencies. [(MATH2101.3, MATH2101.6)(Apply/IOCQ)]

(b) Obtain the equations of the two lines of regression for the given data. Also obtain the estimate of X for $Y = 70$.

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

[(MATH2101.3, MATH2101.6)(Apply/IOCQ]

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	43.75	43.75	12.5

Course Outcome (CO):

After the completion of the course students will be able to

MATH2101.1 Apply numerical methods to obtain approximate solutions to mathematical problems where analytic solutions are not possible.

MATH2101.2 Implement appropriate numerical methods for solving advanced engineering problems dealing with interpolation and integration.

MATH2101.3 Design stochastic models to predict the outcomes of events.

MATH2101.4 Recognize the significance of the expansion of a function in Fourier Series.

MATH2101.5 Provide deterministic mathematical solutions to physical problems through partial differential equations.

MATH2101.6 Employ statistical methods to make inferences on results obtained from an experiment.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.