DISCRETE MATHEMATICS
(CSEN 2102)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) $(p \to q) \leftrightarrow (\neg p \wedge q)$ is a
(a) tautology (b) contradiction (c) contingency (d) none of this.

(ii) "All real numbers are complex numbers"?
Which of the following represents the above statement
(a) $\exists x: [real(x) \to complex(x)]$ (b) $\forall x: [real(x) \wedge complex(x)]$
(c) $\exists x: [real(x) \vee complex(x)]$ (d) $\forall x: [real(x) \to complex(x)]$.

(iii) The value of ${}^{50}C_0\,{}^{50}C_1 + {}^{50}C_1\,{}^{50}C_2 + \dots + {}^{50}C_{49}\,{}^{50}C_{50}$ is:
(a) ${}^{100}C_{50}$ (b) ${}^{100}C_{51}$ (c) ${}^{50}C_{25}$ (d) ${}^{150}C_{25}\,{}^{50}C_{25}$.

(iv) A student can choose a computer project from one of the three lists. The lists contain 23, 15 and 19 possible projects respectively. No project is more than in one list. The number of possible projects to choose from is
(a) 6555 (b) 57 (c) 36 (d) 76.

(v) Total number of non-negative integer valued solutions to the equation $x + y + z = 13$, $x, y, z \geq 0$ is
(a) 39 (b) 105 (c) 120 (d) 286.

(vi) Solution of the recurrence relation $a_n = 2a_{n-1} + 1$ with $a_0 = 0$ is
(a) $2^n - 1$ (b) $2^n - 2$ (c) $2^{n-1} - 1$ (d) $1 - 2^n$.

(vii) What is the Chromatic number of $K_{m,n}$?
(a) 2 (b) m (c) n (d) m+n.

(viii) Given $49x \equiv 1 \pmod{72}$ and $37y \equiv 97 \pmod{125}$, the value of xy =
(a) 925 (b) 54 (c) 150 (d) 180.

(ix) The generating function for the sequence {1, 1, 1, 1, … … . . } is
(a) $\frac{1}{1-x}$ (b) $\frac{1}{(1-x)^2}$ (c) $\frac{1}{(1-x)^3}$ (d) $\frac{1}{(1-x)^4}$.

(x) The chromatic polynomial of a tree with n vertices is
(a) $x(x-1)^{n-1}$ (b) $(x-1)x^{n-1}$ (c) x^n (d) $(x-1)^n$.

Group - B

2. (a) (i) Define Maximum matching and Maximal matching.
(ii) Show an example where a Maximal matching is not a Maximum matching.
[(CO2)(Remember/LOCQ)]

(b) At a chemical company, storage is needed for seven different chemicals. The nature of these chemicals is such that for all 1 ≤ i ≤ 5, chemical 'i' cannot be stored in the same cabinet as chemical 'i+1' or chemical 'i+2'. Determine the smallest number of cabinets needed to store these seven chemicals. [(CO1)(Analyze/IOCQ)]

(c) (i) State Kuratowski's theorem for testing for Planarity, appropriately explaining all terms used.
(ii)

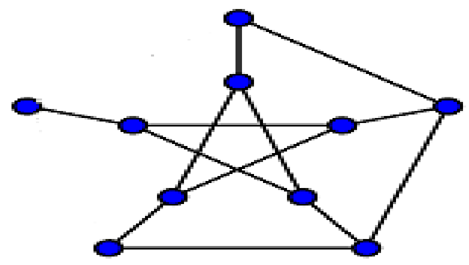

Fig.1

Prove that the above graph in Fig.1 is homeomorphic to $K_{3,3}$.
[(CO1,CO2)(Apply/IOCQ)]

(2 + 2) + 3 + (2 + 3) = 12

3. (a) Prove that a full *m*-ary tree with *l* leaves has *n* = (*ml* – 1)/(*m*-1) vertices and *i* = (*l* – 1)/(*m* – 1) internal vertices. [(CO1)(Remember/LOCQ)]

(b) Does there exists a simple (undirected) graph with the graphic sequence (5, 5, 4, 3, 2, 1)? If yes, draw the graph and if no, explain why. [(CO2)(Understand/LOCQ)]

(c) Take C_5 (cycle having 5 vertices) and K_3 (complete graph of 3 vertices) and join each vertex of C_5 to each vertex of K_3. Find the clique number and the chromatic number of the resulting graph. [(CO2)(Evaluate/IOCQ)]

4 + 2 + 6 = 12

Group - C

4. (a) Let a ≡ b (mod m), c ≡ d (mod m).
Prove that ac ≡ bd (mod m). [(CO3)(Remember/LOCQ)]

(b) Find the gcd of 33 and 27, using the Euclidean algorithm. Using this, find integers x,y such that 33x + 27y = 3. [(CO3)(Understand/LOCQ)]

(c) Find all solutions of x, if they exist, to the system of equivalences:
2x ≡ 6 (mod 14)
3x ≡ 9 (mod 15)
5x ≡ 20 (mod 60). [(CO3)(Analyze/IOCQ)]

3 + (2 + 2) + 5 = 12

5. (a) Prove that 21 divides $4^{n+1} + 5^{2n-1}$ whenever *n* is a positive integer > 0.
[(CO3)(Analyse/HOCQ)]

(b) Which amounts of money can be formed using just two-rupee bills and five-rupee bills? Prove your answer using strong (second principle of) induction.
[(CO3)(Analyze/IOCQ)]

(c) Suppose that m is a positive integer. Use mathematical induction to prove that if a and b are integers with $a \equiv b$ (mod m), then $a^k \equiv b^k$ (mod m) whenever k is a nonnegative integer. [(CO3)(Understand/LOCQ)]

4 + 4 + 4 = 12

Group - D

6. (a) A line separates the plane into two regions. Two intersecting lines separate the plane into four regions. Suppose we have 'n' lines. No two of these lines are parallel. No three lines intersect in the same point.
Find a recurrence relation for the number of regions formed by these 'n' lines (on the plane). [(CO4)(Apply/HOCQ)]

(b) Find a solution to the recurrence relation of the Fibonacci numbers, using the generating function approach. [(CO4)(Understand/LOCQ)]

(c) Consider the linear recurrence relation:

$$a_n = 3\, a_{n-1} + 2^n$$

(i) Show that $a_n = -2^{n+1}$ is a solution of the recurrence relation.

(ii) Find out all solutions of this recurrence relation with $a_0 = 1$.

[(CO4)(Analyze/HOCQ)]

(d) In a examination, the examiner can select at least 2 easy problems, at least 2 medium difficulty problems, and at least 2 hard problems. He has a question bank of 5 easy, 5 medium difficulty and 4 hard problems, from where he can make the selection. the question paper consists of 8 questions. How many ways can he set the question paper? Solve using the generating function approach. [(CO4)(Analyze/IOCQ)]

3 + 3 + 3 + 3 = 12

7. (a) A building inspector has 77 days to complete his duty. He makes at least one inspection a day. He has in total 132 inspections to complete. Is there a period of consecutive days in which he makes exactly 21 inspections? [(CO4)(Analyze/HOCQ)]

(b) There are 10 questions in discrete maths final exam. How many ways are there to assign marks to these questions if the sum of the marks is 100 and each question is worth at least five marks? [(CO4)(Understand/LOCQ)]

(c) How many ways are there to distribute six different toys to three different children such that each child gets at least one toy? [(CO4)(Apply/IOCQ)]

4 + 4 + 4 = 12

Group - E

8. (a) Construct the truth tables of the following statements:

(i) $(p \rightarrow q) \rightarrow (q \rightarrow p)$ (ii) $(\sim p \leftrightarrow q) \leftrightarrow (q \leftrightarrow r)$

(**Note:** ~ denotes the negation of the proposition.) [(CO5)(Understand/LOCQ)]

(b) Define converse, inverse and contra-positive of an implication. Prove that the following pair of proposition is equivalent

$\sim(p \wedge q) \rightarrow (\sim p \vee (\sim p \vee q))$ and $p \rightarrow q$ [(CO6)(Remember/IOCQ)]

(2 + 4) + (2 + 4) = 12

9. (a) State the definitions of conjunctive normal form and disjunctive normal form. [(CO5)(Remember/LOCQ)]

(b) Find the conjunctive normal form (CNF) and disjunctive normal form (DNF) of the following statements:
(i) $p \wedge \sim(q \wedge r) \vee (p \rightarrow q)$ (ii) $p \longrightarrow (p \longrightarrow q) \wedge (\sim (\sim q \vee \sim p))$. [(CO6)(Understand/LOCQ)]

(c) Find the truth value of $'\forall x, P(x)'$ where P(x) is the statement "$x^2 < 20$" and the domain is the set {1, 2, 3, 4}. [(CO5)(Analyse/HOCQ)]

2 + (4 + 4) + 2 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	42	42	16

Course Outcome (CO):

After the completion of the course students will be able to
CSEN2102.1. Interpret the problems that can be formulated in terms of graphs and trees.
CSEN2102.2. Explain network phenomena by using the concepts of connectivity, independent sets, cliques, matching, graph coloring etc.
CSEN2102.3. Achieve the ability to think and reason abstract mathematical definitions and ideas relating to integers through concepts of well-ordering principle, division algorithm, greatest common divisors and congruence.
CSEN2102.4. Apply counting techniques and the crucial concept of recurrence to comprehend the combinatorial aspects of algorithms.
CSEN2102.5. Analyze the logical fundamentals of basic computational concepts.
CSEN2102.6. Compare the notions of converse, contrapositive, inverse etc. in order to consolidate the comprehension of the logical subtleties involved in computational mathematics.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question