# MATHEMATICS - I
## (MATH 1101)

**Time Allotted : 3 hrs** **Full Marks : 70**

***Figures out of the right margin indicate full marks.***

***Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.***

***Candidates are required to give answer in their own words as far as practicable.***

## Group – A
### (Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) If the vector $(x+3y)\hat{\imath}+(y-2z)\hat{\jmath}+(x+mz)\hat{k}$ is solenoidal, then the value of $m$ is
(a) $-1$ (b) 1 (c) $-2$ (d) 2.

(ii) The rank of a sixth order identity matrix is
(a) 1 (b) 0 (c) 36 (d) 6.

(iii) The matrix $A=\begin{pmatrix}\frac{\sqrt{3}}{2} & -\frac{1}{2}\\ \frac{1}{2} & \alpha\end{pmatrix}$ is orthogonal if $\alpha=$
(a) 1 (b) 0 (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$.

(iv) If $\lambda_1=2,\ \lambda_2=-3$ and $\lambda_3=0$ be three eigenvalues of a $3\times 3$ square matrix of $A$, then the value of the determinant of $A$ is
(a) $-1$ (b) $-6$ (c) 0 (d) 1.

(v) The series $\sum\frac{1}{n^p}$ is convergent if
(a) $p\geq 1$ (b) $p>1$ (c) $p<1$ (d) $p\leq 1$.

(vi) If $x=a(\cos\theta+\theta\sin\theta), y=a(\sin\theta-\theta\cos\theta)$, then at $\theta=\frac{\pi}{4}$, we have
(a) $\frac{dy}{dx}=1$ (b) $\frac{d^2y}{dx^2}=\frac{8\sqrt{2}}{a\pi}$ (c) $\frac{dy}{dx}=-1$ (d) $\frac{d^2y}{dx^2}=\frac{-8\sqrt{2}}{a\pi}$.

(vii) The general solution of the differential equation $(y+x)dx+xdy=0$ is
(a) $x^2-y^2=c$ (b) $x^2+2xy=c$
(c) $x^2-xy=c$ (d) $x^2-2xy=c$.

(viii) Integrating factor of the ordinary differential equation $\frac{dy}{dx}+y\cot x=2x\cos x$ is
(a) $\cos x$ (b) $\sin x$ (c) $-\sin x$ (d) $\cot x$.

(ix) The order and degree of the differential equation $\left\{1+\left(\frac{dy}{dx}\right)^4\right\}^{\frac{1}{3}} = \frac{d^2y}{dx^2}$ are

(a) 2, 4 (b) 2, 3 (c) 4, 3 (d) 4, 2.

(x) The value of the line integral $\int_C (dx - xdy)$, where $C$ is the line joining (0,1) to (1, 0)

(a) 0 (b) $-\frac{3}{2}$ (c) $\frac{1}{2}$ (d) $\frac{3}{2}$.

## Group – B

2. (a) Reduce the following matrix $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 1 \end{bmatrix}$ to a row reduced echelon form and hence find its rank. [(MATH1101.1, MATH1101.2)(Understand/LOCQ)]

(b) Find the eigenvalues and the corresponding eigenvectors of the following matrix:

$$A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$ [(MATH1101.1, MATH1101.2)(Apply/IOCQ)]

**6 + 6 = 12**

3. (a) Determine the values of $\lambda$ and $\mu$ for which the system of linear equations

$$x + 2y + 3z = 4$$
$$x + 3y + 4z = 5$$
$$x + 3y + \lambda z = \mu$$

have (i) no solution, (ii) a unique solution and (iii) an infinite number of solutions.

[(MATH1101.1, MATH1101.2)(Analyze /IOCQ)]

(b) State the Cayley-Hamilton theorem. Verify Cayley-Hamilton theorem for

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}.$$ [(MATH1101.1, MATH1101.2)(Evaluate/HOCQ)]

**6 + 6 = 12**

## Group – C

4. (a) Consider a sequence $\{u_n\}$ defined as follows:

$$u_n = \frac{1}{n^2} + \frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \cdots + \frac{1}{(2n)^2}$$

Show that $\{u_n\}$ is monotonically decreasing and bounded.

[(MATH1101.3, MATH1101.4)(Understand/LOCQ)]

(b) Show that $\vec{A} = (6xy + z^3)\hat{\imath} + (3x^2 - z)\hat{\jmath} + (3xz^2 - y)\hat{k}$ is irrotational. Find the scalar function $\varphi$ such that $\vec{A} = \vec{\nabla}\varphi$.

[(MATH1101.3, MATH1101.4)(Understand/LOCQ)]

**6 + 6 = 12**

5. (a) Find the directional derivative of the scalar point function $f(x, y, z) = x^2 + xy + z^2$ at the Point $A(1, -1, -1)$ in the direction $\overrightarrow{AB}$ where $B$ has coordinates $(3, 2, 1)$.

[(MATH1101.3, MATH1101.4)(Remember/LOCQ)]

(b) Discuss the convergence of the following infinite series:

$$\frac{1}{2^3} - \frac{1}{3^3}(1+2) + \frac{1}{4^3}(1+2+3) - \frac{1}{5^3}(1+2+3+4) + \cdots \ldots$$

[(MATH1101.3, MATH1101.4)(Analyze/IOCQ)]

**5 + 7 = 12**

## Group – D

6. (a) Solve $(1+xy)y\,dx + (1-xy)x\,dy = 0$. [(MATH1101.5)(Understand/LOCQ)]

(b) Solve the differential equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = x^2e^{3x}$ using D-operator method.

[(MATH1101.5)(Understand/LOCQ)]

**6 + 6 = 12**

7. (a) Solve: $y = 2px + y^2p^3$, where $p \equiv \frac{dy}{dx}$. [(MATH1101.5)(Understand/LOCQ)]

(b) Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2} + 4y = \sin\ 2x$.

[(MATH1101.5)(Apply/IOCQ)]

**6 + 6 = 12**

## Group – E

8. (a) Evaluate by Green's theorem $\int_\Gamma \{(2xy - x^2)dx + (x+y^2)dy\}$ where $\Gamma$ is the closed curve of the region bounded by $y = x^2$ and $y^2 = x$.

[(MATH1101.6)(Evaluate/HOCQ)]

(b) If $u = u\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, then show that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$.

[(MATH1101.6)(Understand/LOCQ)]

**6 + 6 = 12**

9. (a) By using Euler's theorem on homogeneous function prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$, where $u = \sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$. Hence show that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\frac{\sin u \cos 2u}{4\cos^3 u}$.

[(MATH1101.6)(Remember/LOCQ)]

(b) Change the order of integration to evaluate $\int_0^1 \int_x^{\sqrt{(2-x^2)}} \frac{x}{\sqrt{x^2+y^2}}\,dx\,dy$.

[(MATH1101.6)(Analyze/IOCQ)]

**6 + 6 = 12**

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| Cognition Level | LOCQ | IOCQ | HOCQ |
|---|---|---|---|
| Percentage distribution | 55.21 | 32.29 | 12.5 |

**Course Outcome (CO):**

After the completion of the course students will be able to
MATH1101.1 Apply the concept of rank of matrices to find the solution of a system of linear simultaneous equations.
MATH1101.2 Develop the concept of eigen values and eigen vectors.
MATH1101.3 Combine the concepts of gradient, curl, divergence, directional derivatives, line integrals, surface integrals and volume integrals.
MATH1101.4 Analyze the nature of sequence and infinite series
MATH1101.5 Choose proper method for finding solution of a specific differential equation.
MATH1101.6 Describe the concept of differentiation and integration for functions of several variables with their applications in vector calculus.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.