# MATHEMATICAL METHODS
## (MATH 2001)

**Time Allotted : 3 hrs** **Full Marks : 70**

***Figures out of the right margin indicate full marks.***

***Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.***

***Candidates are required to give answer in their own words as far as practicable.***

## Group – A
### (Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) If $z = r(\cos\theta + i\sin\theta)$, then $|z|^3$ is equal to
(a) $(\cos\theta + isin\,\theta)^3$ (b) $r^3(\cos\theta + isin\,\theta)^3$
(c) $\frac{r^3}{2}$ (d) $r^3$.

(ii) The values of $P_n(1)$ and $P_n(-1)$ are respectively
(a) $1, 0$ (b) $0, -1$ (c) $(-1)^n, 1$ (d) $1, (-1)^n$.

(iii) Particular integral of the partial differential equation:
$$\left(D^2 + DD' - 6D'^2\right)z = \sin(x+y),\ \left(D \equiv \frac{\partial}{\partial x},\ D' \equiv \frac{\partial}{\partial y}\right) \text{ is}$$
(a) $\frac{\cos(x+y)}{4}$ (b) $-\frac{\sin(x+y)}{4}$ (c) $\frac{\sin(x+y)}{4}$ (d) $-\frac{\cos(x+y)}{4}$.

(iv) Complementary function of the partial differential equation
$$\frac{\partial^2 z}{\partial x^2} + (a+b)\frac{\partial^2 z}{\partial x \partial y} + ab\,\frac{\partial^2 z}{\partial y^2} = xy \text{ is}$$
(a) $f_1(y+ax) + f_2(y-bx)$ (b) $f_1(y-ax) + f_2(y-bx)$
(c) $f_1(y+ax) + f_2(y+bx)$ (d) $f_1(y+ax) + f_2(y+bx)$
where $f_1$ and $f_2$ are arbitrary functions.

(v) The value of $\oint_c \frac{z^2-z+1}{z-1}\,dz$, $C$ being $|z| = {}^{e}/_{3}$, is
(a) $2\pi i$ (b) $\frac{1}{2\pi i}$ (c) $0$ (d) $\pi i$.

(vi) For the differential equation $x^2(1-x)y'' + xy' + y = 0$
(a) $x = 1$ is an ordinary point (b) $x = 1$ is a regular singular point
(c) $x = 1$ is an irregular singular point (d) $x = 0$ is an ordinary point.

(vii) $J_{-1/2}(x)$ is
(a) $\sqrt{\frac{2x}{\pi}}\sin x$ (b) $\sqrt{\frac{2}{\pi x}}\sin x$ (c) $\sqrt{\frac{2}{\pi x}}\cos x$ (d) $\sqrt{\frac{\pi}{2x}}\cos x$.

(viii) If $f(z) = \frac{z^2}{(z-1)^2(z+2)}$, then the residue of $f(z)$ at $z = -2$ is

(a) $\frac{5}{9}$ (b) $\frac{4}{9}$ (c) $\frac{1}{9}$ (d) $\frac{3}{9}$.

(ix) The complete solution of $\left(\frac{\partial^2}{\partial x^2} - 2\frac{\partial^2}{\partial x \partial y}\right)z = e^{2x}$ is

(a) $z = \Phi_1(y) + \Phi_2(y+2x) + \frac{e^{2x}}{4}$ (b) $z = \Phi_1(y-2x) + \Phi_2(y+2x) + \frac{e^{2x}}{4}$

(c) $z = \Phi_1(y) + \Phi_2(y-2x) + \frac{e^{2x}}{4}$ (d) $z = \Phi_1(y-2x) + \Phi_2(y+2x) - \frac{e^{2x}}{4}$.

(x) The value of $\oint_C \frac{z^2+1}{z-2}dz$, $C$ being $|z| = \frac{\pi}{3}$ is

(a) 0 (b) $2\pi i$ (c) $\pi i$ (d) $10\pi i$.

## Group – B

2. (a) Show that the function $u(x,y) = \cos x \cosh y$ is harmonic and find its harmonic conjugate. [(MATH2001.1, MATH2001.2)(Apply/IOCQ)]

(b) Use Cauchy's Integral Formula to evaluate $\oint_C \frac{\cos^3 z}{\left(z-\frac{\pi}{4}\right)^3}dz$, where $C$ is the circle $|z| = 1$.
[(MATH2001.1, MATH2001.2)(Evaluate/HOCQ)]

**6 + 6 = 12**

3. (a) Expand $f(z) = \frac{1}{z^2-3z+2}$ in the region (a) $|z| < 1$ (b) $1 < |z| < 2$.
[(MATH2001.1, MATH2001.2)(Understand/LOCQ)]

(b) State the Residue Theorem. Use it to evaluate $\oint_C \frac{z}{(z-1)(z-2)^2}dz$, where $C$ is $|z-2| = \frac{1}{2}$. [(MATH2001.1, MATH2001.2)(Apply/IOCQ)]

**6 + 6 = 12**

## Group – C

4. (a) Find the Fourier series expansion of the following function
$$f(x) = \begin{cases} \pi + 2x, & -\pi < x < 0 \\ \pi - 2x, & 0 \le x \le \pi \end{cases}$$ and hence prove that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$.
[(MATH2001.1, MATH2001.3, MATH2001.4)(Evaluate/HOCQ)]

(b) Find the Fourier sine transform of the function $f(t) = e^{-t}, t > 0$ and hence find the value of $\int_0^\infty \frac{x^2}{(1+x^2)^2}dx$, by using Parseval's Identity.
[(MATH2001.1, MATH2001.3, MATH2001.4)(Apply/IOCQ)]

**7 + 5 = 12**

5. (a) Find the Fourier inverse transform of the function $F(s) = \frac{1}{s^2+6s+25}$.
[(MATH2001.1, MATH2001.3, MATH2001.4)(Apply/IOCQ)]

(b) Find the Fourier transform of the function $f(x) = \begin{cases} x^2, & |x| \le a \\ 0, & |x| > a \end{cases}$
[(MATH2001.1, MATH2001.3, MATH2001.4)(Apply/IOCQ)]

(c) Obtain the half-range Fourier sine series for the function $e^x$ in the interval $0 < x < 1$.
[(MATH2001.1, MATH2001.3, MATH2001.4)(Remember/LOCQ)]
**4 + 4 + 4 = 12**

## Group – D

6. (a) Find the solution in series of the following differential equation
$y'' + (x-1)y' + y = 0$ about the point $x = 0$.
[(MATH2001.5)(Evaluate/HOCQ)]
(b) Express the function $2x^3 + x^2 + 1$ in terms of Legendre polynomials.
[(MATH2001.5)(Remember/LOCQ)]
**8 + 4 = 12**

7. (a) Define generating function for Bessel's function and hence show that
$2nJ_n(x) = x\{J_{n-1}(x) + J_{n+1}(x)\}$ [(MATH2001.5)(Remember/LOCQ)]
(b) Prove that $\int_{-1}^{1} \mathrm{P_m}(\mathrm{x})\mathrm{P_n}(\mathrm{x})\mathrm{dx} = 0$ for $\mathrm{m} \neq \mathrm{n}$. [(MATH2001.5)(Apply/IOCQ)]
**6 + 6 = 12**

## Group – E

8. (a) Obtain a partial differential equation by eliminating the arbitrary functions $f(x)$ and $g(y)$ from $z = yf(x) + xg(y)$. [(MATH2001.1, MATH2001.6)(Understand/LOCQ)]
(b) Solve: $(mz - ny)p + (nx - lz)q = ly - mx$ where $p \equiv \frac{\partial z}{\partial x}, q \equiv \frac{\partial z}{\partial y}$.
[(MATH2001.1, MATH2001.6)(Apply/IOCQ)]
**6 + 6 = 12**

9. (a) Find a complete integral of $px + p^2 = q$ when $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$.
[(MATH2001.1, MATH2001.6)(Apply/IOCQ)]
(b) Solve: $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$ where $u(0, y) = 8e^{-3y}$.
[(MATH2001.1, MATH2001.6)(Evaluate/HOCQ)]
**6 + 6 = 12**

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| Cognition Level | LOCQ | IOCQ | HOCQ |
|---|---|---|---|
| Percentage distribution | 27.1 | 44.8 | 28.1 |

**Course Outcome (CO):**

After the completion of the course students will be able to
MATH2001.1 Construct appropriate mathematical models of physical systems.
MATH2001.2 Recognize the concepts of complex integration, Poles and Residuals in the stability analysis of engineering problems.

MATH2001.3 Generate the complex exponential Fourier series of a function and make out how the complex Fourier coefficients are related to the Fourier cosine and sine coefficients.
MATH2001.4 Interpret the nature of a physical phenomena when the domain is shifted by Fourier Transform e.g. continuous time signals and systems.
MATH2001.5 Develop computational understanding of second order differential equations with analytic coefficients along with Bessel and Legendre differential equations with their corresponding recurrence relations.
MATH2001.6 Master how partial differentials equations can serve as models for physical processes such as vibrations, heat transfer etc.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question.