MATHEMATICS - I
(MATH 1101)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) If A be a square matrix, then $(A - A^T)$ is
(a) symmetric matrix (b) identity matrix
(c) skew-symmetric matrix (d) null matrix.

(ii) Assume $X = [x_1, x_2, \dots, x_n]^T$ is an n-tuple non-zero vector then the $n \times n$ matrix $V = XX^T$
(a) has rank zero (b) has rank one
(c) is orthogonal (d) has rank n

(iii) If the vector field $\vec{F} = (2x^2y + yz)\hat{\imath} + (xy^2 - xz^2)\hat{\jmath} + (axyz - 2x^2y^2)\hat{k}$ be solenoidal, then the value of a is
(a) -5 (b) 6 (c) -6 (d) 0

(iv) Which one of the following is a divergent series?
(a) $\sum_{n=1}^{\infty} \frac{1}{n^4}$ (b) $\sum_{n=1}^{\infty} \frac{1}{n^2}$ (c) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n}}$ (d) $\sum_{n=1}^{\infty} \frac{1}{2^n}$

(v) Integrating factor of $\frac{dx}{dy} + \frac{x}{y \log y} = \frac{2}{y}$ is
(a) x (b) $\log x$ (c) $\log y$ (d) $\log(\log y)$

(vi) Which one of the following equation is exact differential equation?
(a) $(x^2 + 1)dx - xydy = 0$ (b) $xdy + (3x - 2y)dx = 0$
(c) $2xydx + (2 + x^2)dy = 0$ (d) $x^2ydy - ydx = 0$

(vii) If $u = \sin^{-1}\left(\frac{x}{y}\right) + \cos^{-1}\left(\frac{y}{x}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = ?$
(a) $\sin^{-1}\left(\frac{x}{y}\right) + \cos^{-1}\left(\frac{y}{x}\right)$ (b) $2\sin^{-1}\left(\frac{x}{y}\right)$
(c) 0 (d) $\cos^{-1}\left(\frac{y}{x}\right)$

(viii) If $f(x, y) = 0$ then $\frac{dy}{dx}$ is equal to

(a) $-\frac{f_x}{f_y}$ (b) $\frac{f_x}{f_y}$ (c) $\frac{f_y}{f_x}$ (d) $-\frac{f_y}{f_x}$

(ix) The value of $\iint_R dx\,dy$, where R being the triangular region having vertices at (0,0), (2,0) and (1,1) is

(a) 0.5 (b) 1 (c) 0.2 (d) 0.3

(x) The series $\frac{1}{5}+\frac{1}{7}+\frac{1}{9}+\frac{1}{11}+\cdots$ is

(a) convergent (b) divergent
(c) oscillatory (d) conditionally convergent

Group – B

2. (a) Determine the rank of the matrix

$$\begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$$

(b) Evaluate the following determinant by using Laplace's expansion taking minor of order two.

$$\begin{vmatrix} 2 & 0 & 3 & 4 \\ 0 & 1 & 0 & 2 \\ 5 & -1 & 0 & 1 \\ 1 & 0 & 4 & 3 \end{vmatrix}$$

(c) If λ is a non-zero eigen value of a matrix, then show that $1/\lambda$ is an eigen value of A^{-1}.

4 + 5 + 3 = 12

3. (a) Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & -2 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$. Hence find A^{-1}.

(b) Determine the values of a and b for which the system of equation

$$x + 2y + 3z = 6$$
$$x + 3y + 5z = 9$$
$$2x + 5y + az = b$$

has (i) no solution, (ii) unique solution, (iii) infinite number of solutions.

6 + 6 = 12

Group – C

4. (a) Test the convergence of the series:

$1 + \frac{2}{5}x + \frac{6}{9}x^2 + \frac{14}{17}x^3 + \cdots (x > 0)$

(b) Find the directional derivative of the scalar function $f(x,y,z) = x^2 + xy + z^2$ at the point $A(1,-1,-1)$ in the direction of the line AB where B has co-ordinates (3,2,1).

6 + 6 = 12

5. (a) Test the convergence of the infinite series $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n^2+1}$

(b) Find the equation of the tangent plane to the surface $4z = x^2 - y^2$ at the point (3,1,2).

(c) Show that, if $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$, then $\nabla r^n = nr^{n-2}\vec{r}$ where n is a constant.

5 + 4 + 3 = 12

Group – D

6. (a) Obtain the differential equation of the family of parabolas each of which has a latus rectum $4a$ and whose axes are parallel to the x-axis.

(b) Solve the following differential equation by using the method of variation of parameter $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{1}{1+e^{-x}}$.

6 + 6 = 12

7. (a) Solve: $3x^4p^2 - xp - y = 0$

(b) Solve the following differential equation by using D- operator $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$.

6 + 6 = 12

Group – E

8. (a) Verify Green's theorem in a plane for $\oint_C (y - \sin x)\,dx + \cos x\,dy$, where C represents the triangle with vertices (0,0), $(\pi/2, 0)$, $(\pi/2, 2)$.

(b) Show that $\lim_{(x,y)\to(0,0)} \frac{x^2y^4}{(x^2+y^4)^2}$ does not exists.

(c) If $x = uv$, $y = \frac{u-v}{u+v}$, then find $\frac{\partial(u,v)}{\partial(x,y)}$.

6 + 3 + 3 = 12

9. (a) Change the order of integration $\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$ and hence evaluate.

(b) If $u = (x^2 + y^2)^{2/3}$, prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{4}{9}u$.

6 + 6 = 12

Department & Section	Submission Link
BACKLOG ALL	https://classroom.google.com/c/MjA0OTk4NDc2MDI0/a/MjY1MTk5MjkwMTcx/details