DIGITAL SIGNAL PROCESSING (ELEC 3141)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) A signal $x(t) = 2\cos(40\pi t) + \sin(60\pi t)$ is sampled at 75 Hz. The nature and common period of the sampled signal $x(n)$ are
(a) Periodic ; $N = 15$ (b) Periodic; $N = 5$
(c) Non-periodic; $N = 20$ (d) Periodic; $N = 10$.

(ii) A sequence of operations such as "down-sampling (with $M = 2$) and then up-sampling (with $L = 2$) is performed on a given signal $x(n) = \{1, 4, 6, 8, 0\}$; $n = 0\ to\ 4$ using linear interpolation. For such sequence of operations, a new signal $y(n)$ is generated as
(a) $y(n) = \{1, 5, 4, 3, 2\}$; $n = 0\ to\ 4$ (b) $y(n) = \{1, 2, 5, 3, 1\}$; $n = 1\ to\ 5$
(c) $y(n) = \{1, 3.5, 6, 3, 0\}$; $n = 0\ to\ 4$ (d) $y(n) = \{1, 3.5, 6, 3, 0\}$; $n = -1\ to\ 3$.

(iii) Let the convolution sum between the two signals $x(n) = \{3, 1, 4\}\ for\ n = 0, 1, 2$; and $h(n) = \{1, 2\}\ for\ n = 0, 1$ is $y(n) = \{3, 7, 6, 8\}$; $n = 0, 1, 2, 3$. The convolution sum of $z(n) = 2x(n-2) * h(n)$ is given by
(a) $z(n) = \{1.5, 3.5, 3, 4\}$; $n = 2, 3, 4, 5$ (b) $z(n) = \{3, 7, 6, 8\}$; $n = -2, -1, 0, 1$
(c) $z(n) = \{6, 14, 12, 16\}$; $n = 2, 3, 4, 5$ (d) $z(n) = \{6, 14, 12, 16\}$; $n = 0, 1, 2, 3$.

(iv) If the $z-$ transform of a signal $x(n) = u(n)$; $n \geq 0$ is $X(z) = \frac{z}{z-1}$; ROC: $|z| > 1$, then $z - transform$ of $\bar{x}(n) = nx(n)$ is
(a) $\bar{X}(z) = \frac{1}{z(z+1)}$; $ROC: |z| < 1$ (b) $\bar{X}(z) = \frac{1}{z(z-1)^2}$; $ROC: |z| > 1$
(c) $\bar{X}(z) = \frac{1}{z(z-1)}$; $ROC: |z| < 1$ (d) $\bar{X}(z) = \frac{z}{(z-1)^2}$; $ROC: |z| > 1$.

(v) A discrete-time system is excited by an input signal $x(n) = 2(0.4)^n u(n)$ and with an initial condition, the corresponding "zero state (zs) response" and "zero input (zin) response" are noted as $y_{zs}(n) = (4(0.8)^n - 2(0.4)^n)\, u(n)$ and $y_{zin}(n) = -8(0.8)^n u(n)$ respectively. The complete output response $y(n)$ of the system due to the input signal $x_1(n) = 3 \times x(n) = 3 \times 2(0.4)^n$ and with the same initial condition is given by

(a) $y(n) = \{(0.4)^n - 5(0.8)^n\}u(n)$ (b) $y(n) = \{-(0.4)^n - 10(0.8)^n\}u(n)$
(c) $y(n) = \{-4(0.4)^n + 8(0.8)^n\}u(n)$ (d) $y(n) = \{4(0.8)^n - 6(0.4)^n\}u(n)$.

(vi) Let $X(k) = \left\{\boldsymbol{a},\ -\frac{1}{2} - j\,\frac{\sqrt{3}}{2},\ -\frac{1}{2} + j\,\frac{\sqrt{3}}{2}\right\}$ (for $k = 0\ to\ 2$) is a 3-point DFT sequence of $x(n)$ and the normalized energy of the signal $\sum_{n=0}^{2}|x(n)|^2 = 6$. The value of "$X(0) = a$" is

(a) 2 (b) 5 (c) 4 (d) 3.

(vii) If the DFT coefficients of 4-point sequence of a signal $x(n)$ is $X(k) = \{4, -j2,\ 0,\ j2\}$; $k = 0\ to\ 3$, then DFT of $g(n) = x(-n)$ is

(a) $G(k) = \{1, j4, 2, -j4\}; n = 0\ to\ 3$ (b) $G(k) = \{0, j2, 4, -j2\};\ n = 0\ to\ 3$
(c) $G(k) = \{4, j2, 0, -j2\}; n = 0\ to\ 3$ (d) $G(k) = \{1, -j, 4, -j\}; n = 0\ to\ 3$.

(viii) If, a causal and stable discrete time system $H(z) = \frac{z}{z+0..2}$ is excited with a sinusoidal input $x(n) = \cos(0.05\pi n)\,u(n)$ then the percentage of input power signal transmitted through the system $(H(z))$ at the output is

(a) 60.0% (b) 69.8% (c) 89% (d) 78.5%.

(ix) If the $z -$ transform of a signal $(x(n))$ is $X(z) = \frac{z}{z-0.5}$; ROC: $|z| > 0.5$, then $z - transform$ of $\bar{x}(n) = nx(n)$ is

(a) $\bar{X}(z) = \frac{1}{z(z-0.5)}; ROC:\ |z| < 0.5$ (b) $\bar{X}(z) = \frac{(z-1)}{(z-0.5)}; ROC: 0.5 < |z| < 1$
(c) $\bar{X}(z) = \frac{(z-0.5)}{1}; ROC:\ |z| > 0.5$ (d) $\bar{X}(z) = \frac{0.5z}{(z-0.5)^2}; ROC:\ |z| > 0.5$.

(x) Let the location of a zero of a linear-phase $Filter$ is at $z = 0.5\sqrt{3} + j0.5$. The transfer function $H(z)$of minimum length linear phase FIR filter having even length and odd symmetry impulse response $h(n)$ is

(a) $H(z) = 1 - 2.732z^{-1} + 2.732z^{-2} - z^{-3}$
(b) $H(z) = 1 + 2.732z^{-1} - 2.732z^{-2} + z^{-3}$
(c) $H(z) = 1 - 2.732z^{-1} + 2.732z^{-2} + 5z^{-4}$
(d) $H(z) = 1 - 2.32z^{-1} - 2.32z^{-2} + 1.z^{-5}$.

Group – B

2. (a) Find the energy or power of the signal described by) $x(n) = (0.6)^{|n|}\cos\left(\pi\frac{n}{2}\right)$.

(b) Is the discrete-time system $y(n) = 10\,x(n)\cos\,(0.25\pi n + \theta)$ is linear and stable (where $y(n)$ = output signal, $x(n)$= input signal and θ is a constant)?

(c) Find the linear convolution of the sequences $x(n) = \{3,\ 2,\ 4\};\ n = 0,\ 1,\ 2$ and $h(n) = \{-2,\ 3\};\ n = 0,\ 1$ **using graphical method**. If $x(n)$ and $h(n)$ are the input and impulse response of the system, obtain the output of the system $y(n)$ for $n = 0,\ 1,\ 2,\ 3$. Write the possible difference equation of the system.

4 + 2 + (5 + 1) = 12

3. (a) An impulse response of a system is given as $h(n) = 1,\ 2,\ 2,\ 3\}$. Find the output response of the system corresponding to an input signal $x(n-2)$ where the signal $x(n)$ is defined as $x(n) = \{2, -1,\ 3\}$ for $n = 0,\ 1,\ 2$. Sketch output response-vs- time index (n).

(b) Check the stability of the system from its impulse response $h(n) = (0.8)^{-n}u(-n-1) + (0.8)^{-n}u(n)\ for\ n = -\infty\ to\ \infty$, using the stability criterion $\sum_{n=-\infty}^{\infty}|h(n)| < \infty$.

6 + 6 = 12

Group – C

4. (a) Find the $z-$ transform of the signal

$x(n) = (0.8)^{-n}u(-n-1) + (0.8)^{-n}u(n)\ for\ n = -\infty\ to\ \infty$, and its $R.O.C$.

(b) Find the solution of difference equation $y(n) = 0.5y(n-1) + 0.5(0.2)^n u(n)$ using the z-transform method when the initial condition is given by $y(-1) = 1$.

6 + 6 = 12

5. (a) Transform the analog filter system $H_c(s) = \frac{s+1}{s^2+5s+6}$ into a digital filter system $H(z)$ using "$forward\ difference$" –transformation, assuming the sampling time $T = 1$s.

(b) Realize the following digital filter
$y(n) = \frac{5}{4}y(n-1) - \frac{3}{4}y(n-2) + \frac{1}{8}y(n-3) + 8x(n) - 4x(n-1) - 2x(n-3)$
using "direct-form-II" structure. Is the filter stable?

6 + 6 = 12

Group – D

6. (a) Find the signal $x(n)$ from the four DFT coefficient values
$X(k) = \{6, -2+j2, -2, -2-j2\}$ for $k = 0\ to\ 3$ using inverse DFT method.

(b) For each DFT pair shown,

$\{x(0), 3, -4, 0, 2\} \Leftrightarrow \{5, X(1), -1.28 - j4.39, X(3), 8.78 - j1.4\}$

Compute the values of the boxed quantities in the sequences. Justify your answer.

(c) State the Parseval's relation and explain with the numerical example given in Q6(b).

6 + 3 + 3 = 12

7. (a) Given the 4-point DFT coefficients as $X(k) = \{1,\ 2-j,\ -1,\ 2+j)\ for\ k = 0\ to\ 3$, evaluate its inverse DFT $x(n)$ using $DIT - FFT$ method.

(b) Let $x_1(n) = \{-1,\ 2, -5\}; n = 0\ to\ 2$ and $x_2(n) = \{2, -1,\ -2\}; n = 0\ to\ 2$. Find the circular convolution between the signals $x_1(n-2)$ and $x_2(n)$ i.e. $x_1(n-2) \otimes x_2(n)$.

(c) Why we care about "circular convolution" and how it is different from the "linear convolution"?

6 + 5 + 1 = 12

Group – E

8. (a) Consider a periodic signal $x(n)$ = sin(0.1πn) + $\frac{1}{5}$(0.5πn) is communicated through a channel or system whose impulse response $h(n)$ = {1, −0.176, 1 ; n=0, to 2. Determine if the channel or system imparts any (i) magnitude "distortion" (ii) "phase distortion".

(b) The transfer function of discrete-time filter is given as,

$$H(z) = 2 + z^{-1} - z^{-3} - 2z^{-4}.$$

Is this filter is linear-phase filter and its type? What is the delay in the response of the filter? Locate the poles and zeros of $H(z)$.

(c) A low-pass ***FIR*** filter having the following frequency specifications:
Pass band edge frequency Ω_p = 0.375$\pi\ rad/s$; stopband edge frequency Ω_s = 0.5$\pi\ rad/s$; cutoff frequency Ω_c = 0.438$\pi\ rad/s$; transition band(width) = 0.125$\pi\ rad/s$; passband ripple $\delta_p \le 0.0575$; stopband $\delta_s \le 0.0032$. Sketch the tolerance diagram for the low-pass-filter indicating all design specifications.

4 + 4 + 4 = 12

9. (a) Consider a discrete-time "all-pass" filter having two poles $p_{1,2} = 1 \pm 2j$, find the transfer function of the system? Sketch the pole and zero locations of the system.

(b) Find the low-frequency (or D.C) and high frequency gains of the system $H(z) = \frac{(1+5.657z^{-1}+16z^{-2})}{(1-0.8z^{-1}+0.64z^{-2})}$. Is the system is minimum/ non-minimum/ mixed system or all-pass system?

(c) The transfer function of digital filter $H(z)$ is given by

$$H(z) = \frac{(0.0736 - 0.2208z^{-1} + 0.2082z^{-2} - 0.0736z^{-3})}{(1 + 0.9761z^{-1} + 0.8568z^{-2} + 0.2919z^{-3})}$$

Check whether the given filter will act as a low-pass or high-pass filter (without calculating the magnitude of the frequency response of the filter) by calculating the filter gains only at low frequency and high frequency-Justify the answer?

3 + 3 + 6 = 12

Department & Section	Submission Link
EE	https://classroom.google.com/c/MTIyMTkyOTEyMTk4/a/MjcxNDgyNzUxNjky/details