A multi-item inventory model with random replenishment intervals, fuzzy costs and resources under Possibility and Necessity Measure

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Abstract

This paper presents a multi-product multi-period inventory problem with random replenishment intervals and fuzzy costs under space and shortage level constraints. Since the costs (purchasing, holding and backordering) related to inventory system are often imprecise and the replenishment intervals are random in real life, the proposed model is also fuzzy-random Here, the replenishment intervals are taken to be i.i.d random variables and the fuzzy costs are Trapezoidal fuzzy number (TrFN). Using the probability distribution between replenishment epochs the fuzzy-random model is transformed to a fuzzy expected profit model and then using fuzzy arithmetic under function principle the optimistic and pessimistic values of the objective function are obtained. The optimum order quantities for maximum profit are determined with the help of Generalised Reduced Gradient (GRG) method. To illustrate the solution procedure a numerical solution is provided.

Keywords: Replenish-up-to inventory control, Random replenishment interval, Possibility and Necessity measures, m_ρ - measure

1. Introduction

In multi-period inventory control models, continuous review and periodic review are the two main policies. In case of the first policy order can be made at any time depending on the inventory position, and in the second policy an order can be initiated only at the beginning of each period.

Nahmias (1971) considered a 'periodic review inventory model' with lost sale, partial backlogging and random lead times under no order crossing assumption. He solved the model by using two heuristics. Donselaar et al. (1996) also suggest another heuristic to

find order-up-to level in a periodic review system allowing lost sales. Qu et al. (1999) investigated an integrated inventory-transportation system for multiple products. Downs et al. (2001) developed an inventory problem with multiple items, resource constraints, lags in delivery and lost sales. After showing the convexity of the inventory costs in order-up-to level, they develop a linear programming model based on non parametric estimates of these costs. Chiang (2003) studied a periodic review inventory system with long review periods. He employed a dynamic programming approach to model the problem. Chiang (2006) also considered a periodic review inventory system with replenishment cycles that consists a number of periods. Eynan and Kropp (2007) proposed a periodic review system with the assumption of stochastic demand, variant warehousing cost and safety stock. Teunter et al (2010) proposed a method for determining order-up-to levels under periodic review for compounded binomial demand. Recently, Bijvan and Johansen (2012) proposed a periodic review lost sales inventory models with compounded Poisson demand and constant lead times.

Nahmias and Demmy (1981) were also the first researchers to considered stock rotating in an (s, S) policy under static rotating in continuous review environment. They assume two demand classes with unit Poisson arrivals, constant lead time and full backordering for performance evaluation process. Moon and Kang (1998) considered the compound Poisson demand arrivals and provide a simulation study on the setting of Nahmias and Demmy (1981). Moon and Cha (2005) investigated a continuous review inventory model under the assumption that the replenishment lead time depends on lot size and the production rate of the manufacturer. Jeddi et al. (2004) developed a multi product continuous review system with stochastic demand and shortages under budgetary constraint. Mohebbi and Posner (2002) considered a continuous review inventory system for multiple replenishment orders with lost sales. Taleizadeh (2008) developed a multi product, multi constraints inventory model with stochastic replenishment. They showed that the model to be an integer non-linear programming and proposed a Simulated Annealing to solve it. Chiang (2010) considered an order expediting policy for continuous review systems with manufacturing lead time. Recently, Axsater and Viswanthan (2012) proposed a continuous review inventory problem of an independent supplier to evaluate the value of information about the customer's inventory level.

In most of the existing literature, inventory related costs are assume to be deterministic and represented as real numbers. But, in real situation the inventory costs are usually imprecise in nature due to the influence of various uncontrollable factors. For example, costs may depend on some foreign monetary unit. In such a case, due to exchange rates, the costs are often not known precisely. Inventory carrying cost may also dependent on some parameters like interest rate and stock keeping unit's market price, which are not precise. Also the shortage cost is often difficult to determine precisely in the case when it reflects not just 'lost sale' but also 'a loss of customers will'. Therefore, these cost parameters are described as "approximately equal some certain amount" and so it is more reasonable to characterize these parameters as fuzzy.

Since such type of uncertainties cannot be measured properly using the concept of probability theory, fuzzy set theory has been used to model the real uncertain inventory situation. Park (1987) applied fuzzy set theory to classical EOQ model by representing ordering and holding costs with fuzzy numbers and solved by fuzzy arithmetic operation based extension principle. Chen and Wang (1996) fuzzified the demand,

ordering cost, carrying cost and backorder cost into Trapezoidal fuzzy numbers in EOQ model with backorder. Petrovic et al. (1996) developed a newsboy problem in fuzzy environment where uncertain demand was represented by a discrete fuzzy set and inventory cost was given as triangular fuzzy number. Yao and Lee (1999), Yao and Su (2000) and Yao and Chiang (2003) discussed various inventory problems without and with backorder and production inventory control. Besides, some researchers incorporate chance constraint programming introduced by Liu and Iwamura (1998) in inventory models. Maiti and Maiti (2006) extended this work where pessimistic return of the objective function is optimized using necessity measure of fuzzy event and they used to solve a two-warehouse fuzzy inventory model. Wang et al. (2007) proposed fuzzy dependent chance programming model to find the optimal order quantity for maximizing the credibility of an event such that total cost in playing periods does not exceed a certain budget level. Chiang (2010) developed a single item continuous review order expediting inventory policy with manufacturing lead time. Dey and Chakraborty (2012) proposed a periodic review inventory system with variable lead time and negative exponential crashing cost in fuzzy-random environment. Recently, Wang et al (2012) considered two continuous review inventory models with backorders and lost sales under fuzzy demand and different demand situations.

To the best of our knowledge the past works on fuzzy inventory model considered either optimistic or pessimistic approach. If the decision maker (DM) is optimistic, he/she may choose possibility measure. According to Gao and Liu (2001) a fuzzy event may fail even though its possibility achieves 1, and hold even though its necessity is zero. Consequently, high level of confidence in possibility measure does not guarantee the occurrence of fuzzy event. However, the fuzzy event must hold if its credibility is 1, and fail if its credibility is zero. The viewpoint of this research work is different which considers the DM to be eclectic. Therefore we need to make use a combination of both possibility and necessity measure.

In this paper, a multi-product inventory model with space constraint and shortage level constraint is formulated in random fuzzy environment. Here the time periods between replenishments are stochastic variables and follows exponential distribution with a known mean and inventory costs are imprecise and represented by trapezoidal fuzzy numbers (TrFN). The rest of the paper is organized as follows. Section 2 provides a brief introduction to the possibility and necessity measures. Section 3 presents notation, problem assumptions and the proposed problem formulation. In section 4, considering optimistic and pessimistic values of the objective function two methods are suggested for solving the problem. Sections 5 provide a numerical example and section 6 discuss the results. The conclusion and future scope is given in section 7.

2. Basic concept and methodology

In this section, we introduce some basic concepts of possibility, necessity measures of a fuzzy event.
To measure the possibility that a fuzzy set belongs to another fuzzy set, we need to introduce the definition of possibility and necessity measures. The definitions are given as follows:

Definition 2.1: Suppose ξ is restricted by a fuzzy set $\tilde{A}$ in the universe X. Further suppose that the possible distribution of ξ, π_ξ is taken to be equal to the membership function $\mu_{\tilde{A}}(x)$. Then the possibility of the fuzzy event $\{\xi \in \tilde{B}\}$ can be defined by

$$\text{Pos}\{\xi \in \tilde{B}\} = \sup_{x \in X} \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}.$$

The dual measure of possibility, i.e. the necessity measure of the event $\{\xi \in \tilde{B}\}$ is defined as

$$\text{Nec}\{\xi \in \tilde{B}\} = \inf_{x \in X} \max\{1 - \mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}.$$

Suppose b is a crisp number, then $\text{Pos}\{\tilde{A} \leq b\}$ represent the maximum likelihood of the event that $\tilde{A}$ is less than b and $\text{Nes}\{\tilde{A} \leq b\}$ estimates the minimum likelihood of the event that $\{\tilde{A} \leq b\}$ will occur. By definition, we have

$$\text{Pos}\{\tilde{A} \leq b\} = \text{Pos}\{\xi \in (-\infty, b]\} = \sup_{x \leq b}\{\mu_{\tilde{A}}(x)\}$$

$$\text{Nes}\{\tilde{A} \leq b\} = \text{Nes}\{\xi \in (-\infty, b]\} = \inf_{x > b}\{1 - \mu_{\tilde{A}}(x)\}$$

$$\text{Pos}\{\tilde{A} \geq b\} = \text{Pos}\{\xi \in [b, \infty)\} = \sup_{x \geq b}\{\mu_{\tilde{A}}(x)\}$$

$$\text{Nes}\{\tilde{A} \geq b\} = \text{Nes}\{\xi \in [b, \infty)\} = \inf_{x < b}\{1 - \mu_{\tilde{B}}(x)\}.$$

Example 2.1: A trapezoidal fuzzy variable $\tilde{A}$ determined by quadruplet (a_1, a_2, a_3, a_4) of crisp numbers with $a_1 < a_2 < a_3 < a_4$, whose membership function is given by (cf. Fig.-1)

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & \text{if } a_1 \leq x < a_2 \\ 1 & \text{if } a_2 \leq x \leq a_3 \\ \dfrac{a_4 - x}{a_4 - a_3} & \text{if } a_3 \leq x \leq a_4 \\ 0 & \text{otherwise} \end{cases}$$
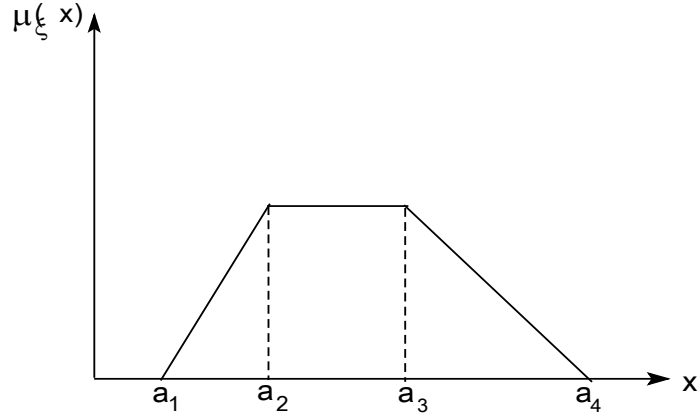
Figure 1. Membership function of trapezoidal fuzzy (TrFN) number

According to Definition 2.1, we can easily obtain the possibility and necessity of fuzzy event $\{\tilde{A} \geq x\}$ as

$$\text{Pos}\{\tilde{A} \geq x\} = \begin{cases} 1 & \text{if } x \leq a_3 \\ \dfrac{a_4 - x}{a_4 - a_3} & \text{if } a_3 < x \leq a_4 \\ 0 & \text{if } x > a_4, \end{cases}$$

$$\text{Nec}\{\tilde{A} \geq x\} = \begin{cases} 0 & \text{if } x \geq a_2 \\ \dfrac{a_2 - x}{a_2 - a_1} & \text{if } a_1 \leq x < a_2 \\ 1 & \text{otherwise,} \end{cases}$$

In fuzzy inventory models, possibility and necessity measures are employed by many researchers. Since fuzzy estimates are based on human judgement, however, they should reflect some assessment of whether the DM tends towards a 'looser' interpretation of fuzzy estimate (possibility) or a 'tighter' one (necessity). Actually, for the optimistic DM, the possibility measure is much suitable where as if the DM is pessimistic; he may use the necessity measure as a tool to make the decision.

According to Yang and Iwamura [2008], if a DM wants to seek the best decision to maximize the chance of fuzzy event $\{f(\mathbf{x}, \tilde{\mathbf{A}}) \in \mathbf{B}\}$, $\mathbf{x} = (x_1, x_2, \ldots, x_n)^T$ is the decision variables vector, $\tilde{\mathbf{A}}$ is fuzzy parameter vector and $\mathbf{B} \in \mathbf{R}^n$ (n-dimensional real space). If the DM use possibility measure as a chance measure then a decision $\mathbf{x}^*$ will be recognised as the best and it satisfies $\text{Pos}\{f(\mathbf{x}, \tilde{\mathbf{A}}) \in \mathbf{B}\} = 1$. In fact, a fuzzy event may fail though its possibility may achieve 1. This implies that for some realization value $\mathbf{y}$

with $\mu_{\tilde{A}}(y) > 0$, the event $\{ f(x^*, y) \in \mathbf{B} \}$ may not appear. So depending upon the nature of the DM, he/she may choose the possibility measure if he/she is optimistic and does not care about the potential risk otherwise he/she may choose necessity measure as a chance measure. In practice, the decision $\mathbf{x}^*$ is not necessarily the best decision for the necessity measure since the corresponding objective value is less than or equal to 1. Thus the DM may select a better solution $\overline{\mathbf{x}}^*$ as the optimal decision. If the necessity measure of fuzzy event $\{ f(\overline{\mathbf{x}}^*, \tilde{A}) \in \mathbf{B} \}$ achieves 1, the realization value $\overline{\mathbf{x}}$ of $\tilde{A}$ with $\mu_{\tilde{A}}(\overline{\mathbf{x}}) > 0$, the event $\{ f(\overline{\mathbf{x}}^*, \tilde{A}) \in \mathbf{B} \}$ must hold.

In practice, most DMs are neither absolutely optimistic nor absolutely pessimistic. Accordingly, a DM attitude factor ρ $(0 \le \rho \le 1)$ was introduced in the decision process by Wang and Shu [2005], which produce a balance between the optimistic and the pessimistic.

Suppose, $\overline{m} = \text{Pos}\{ \tilde{A} \le b \}$ and $\underline{m} = \text{Nes}\{ \tilde{A} \le b \}$; then the weighted DM's judgement of the event $\{ \tilde{A} \le b \}$ is given by
$$m_\rho = \rho\,\overline{m} + (1 - \rho)\,\underline{m},$$
where ρ is predetermined by the DM according to his nature. Further larger value of the parameter ρ, the DM is more optimistic and if $\rho = 1$, then m_ρ - measure degenerates to possibility measure. If $\rho = 0$, then m_ρ - measure degenerates to necessity measure. If $\rho = 0.5$, then m_ρ - measure degenerates to credibility measure by Liu and Liu [2002].

Definition 2.2: If $\tilde{A}$ be a fuzzy variable and $\alpha \in (0, 1]$, then
$$\tilde{A}_{\inf}(\rho, \alpha) = \inf\{ b : m_\rho\{ \tilde{A} \le b \} \ge \alpha \} \text{ and}$$
$$\tilde{A}_{\sup}(\rho, \alpha) = \sup\{ b : m_\rho\{ \tilde{A} \ge b \} \ge \alpha \}$$
are respectively called the (ρ, α) - pessimistic and (ρ, α) – optimistic values of $\tilde{A}$.

Lemma 2.1: If a TrFN $\tilde{A}$ determined by quadruplet (a_1, a_2, a_3, a_4) then we have
$$\tilde{A}_{\inf}(\rho, \alpha) = \begin{cases} \dfrac{\alpha(a_2 - a_1)}{\rho} + a_1 & \text{if } \alpha \le \rho \\[2mm] \dfrac{(1 - \alpha)(a_3 - a_4)}{(1 - \rho)} + a_4 & \text{if } \alpha > \rho \end{cases}$$
and
$$\tilde{A}_{\sup}(\rho, \alpha) = \begin{cases} \dfrac{\alpha(a_3 - a_4)}{\rho} + a_4 & \text{if } \alpha \le \rho \\[2mm] \dfrac{(1 - \alpha)(a_2 - a_1)}{(1 - \rho)} + a_1 & \text{if } \alpha > \rho \end{cases}$$

Proof:
$$m_\rho\{ \tilde{A} \le b \} = \rho\,Pos\{ \tilde{A} \le b \} + (1 - \rho)\,Nes\{ \tilde{A} \le b \}$$

$$= \rho\, Pos\{\tilde{A} \le b\} + (1-\rho)\left(1 - Pos\{\tilde{A} \ge b\}\right)$$

$$= \begin{cases} 0 & \text{if } b < a_1 \\ \rho\left(\dfrac{b - a_1}{a_2 - a_1}\right) & \text{if } a_1 \le b < a_2 \\ \rho & \text{if } a_2 \le b < a_3 \\ \rho + (1-\rho)\left(\dfrac{a_4 - b}{a_4 - a_3}\right) & \text{if } a_3 \le b \le a_4 \\ 1 & \text{if } a_4 < b \end{cases}$$

This is easy to see that

$$\tilde{A}_{\inf}(\rho, \alpha) = \begin{cases} \dfrac{\alpha(a_2 - a_1)}{\rho} + a_1 & \text{if } \alpha \le \rho \\ \dfrac{(1-\alpha)(a_3 - a_4)}{(1-\rho)} + a_4 & \text{if } \alpha > \rho \end{cases}$$

and

$$\tilde{A}_{\sup}(\rho, \alpha) = \begin{cases} \dfrac{\alpha(a_3 - a_4)}{\rho} + a_4 & \text{if } \alpha \le \rho \\ \dfrac{(1-\alpha)(a_2 - a_1)}{(1-\rho)} + a_1 & \text{if } \alpha > \rho \end{cases}$$

3. Constraint Fuzzy-Random Inventory Model

The mathematical model in this paper is developed on the basis of the following assumptions and notations:

3.1 Assumptions

To describe the problem we introduce the following assumptions:

(1) The times between replenishments are i.i.d random variables.
(2) Demand rate is known and constant.
(3) Inventory costs (purchasing, holding and shortage) are not known precisely and represents as Trapezoidal fuzzy numbers (TrFN).
(4) Lead time is zero.
(5) Shortages are allowed, but backlogged partially.

3.2 Notations

The following notations are employed throughout this paper to develop this model

PF   total expected profit
W   total available warehouse space

For i^{th} (i = 1, 2, ….., n) product

D_i: demand rate

Q_i: initial inventory level

R_i: expected amount order in each cycle

$\hat{T}_i$:   time period between two replenishment (a random variable)

T_{Maxi} upper limit of the probability distribution of $\hat{T}_i$

T_{Mini} lower limit of the probability distribution of $\hat{T}_i$

$f_{Ti}(t_i)$ probability density function of $\hat{T}_i$

T_{0i}:   time at which inventory level reaches zero

β_i: fraction of unmet demand backordered

s_i: sales price per unit item

$\tilde{h}_i$:   holding cost per unit quantity per unit time (a fuzzy variable)

$\tilde{\pi}_i$:   shortage cost per unit quantity per unit time (a fuzzy variable)

$\tilde{p}_i$:   purchase cost per unit quantity of material (a fuzzy variable)

S_i: lower limit of the service level

w_i: required warehouse space per unit item.

3.3 Model formulation

In the development of the inventory model of the ith product, we assume the time periods between replenishments are stochastic variables. According to Ertogal and Rahim (2005) two cases may occur, in the first case the time between replenishments is less than the amount of time required for the inventory level depleted completely (Fig.-2) and in the second case, the time between replenishment exceeds the period in which the inventory level depletes zero and shortage occurs (Fig.-3) which are backlogged partially at the beginning of each period.
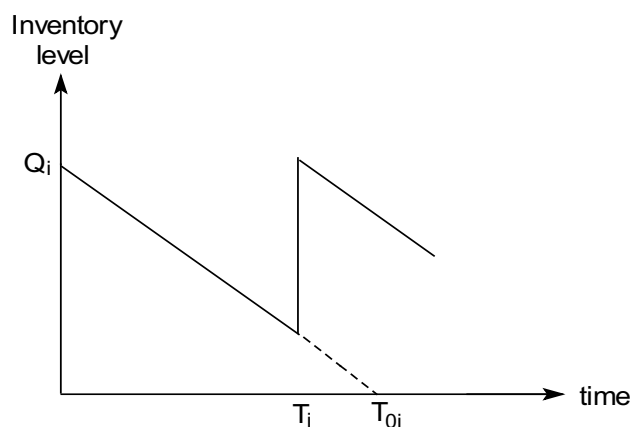

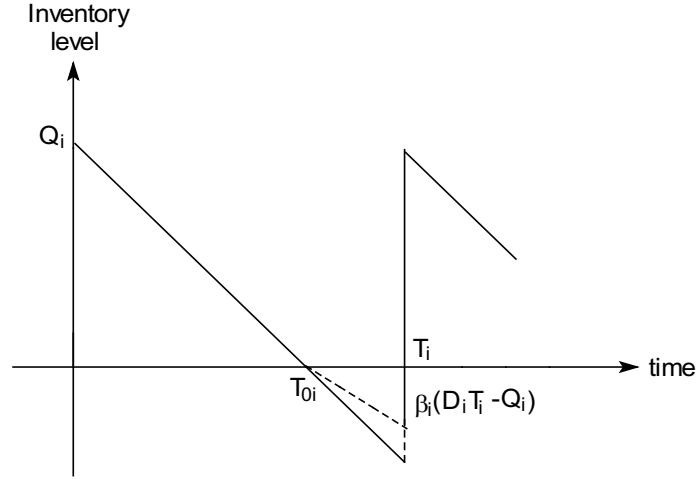Figure 2. Presenting the inventory cycle for no backorder case

Figure 3. Presenting the inventory cycle for backorder case

To calculate the expected profit per cycle for the ith product in fuzzy random environment, we need to evaluate the following:

$$PF(Q_i, \hat{T}_i, \tilde{p}_i, \tilde{h}_i, \tilde{\pi}_i) = (s_i - \tilde{p}_i)R_i - \tilde{h}_i I_i - (s_i - \tilde{p}_i)L_i - \tilde{\pi}_i B_i$$

where

$$R_i = \int_{T_{Min_i}}^{t_{0i}} D_i T_i f_{T_i}(t_i) dt_i + \int_{t_{0i}}^{T_{Max_i}} \left\{ Q_i + \beta_i D_i \left(T_i - \frac{Q_i}{D_i} \right) \right\} f_{T_i}(t_i) dt_i$$

The expected average inventory in a cycle is

$$I_i = \int_{T_{Min_i}}^{t_{0i}} \left(Q_i T_i + \frac{D_i T_i^2}{2} \right) f_{T_i}(t_i) dt_i + \int_{t_{0i}}^{T_{Max_i}} \left(\frac{Q_i^2}{2D_i} \right) f_{T_i}(t_i) dt_i$$

The expected total unmet demand in a cycle is

$$B_i = \beta_i \int_{t_{0i}}^{T_{Max_i}} (D_i T_i - Q_i) f_{T_i}(t_i) dt_i$$

and the expected lost demand in a cycle is

$$L_i = (1 - \beta_i) \int_{t_{0i}}^{T_{Max_i}} (D_i T_i - Q_i) f_{T_i}(t_i) dt_i$$

Then the total expected profit for all products is as follows

$$PF(\boldsymbol{Q}, \hat{\boldsymbol{T}}, \tilde{\boldsymbol{p}}, \tilde{\boldsymbol{h}}, \tilde{\boldsymbol{\pi}}) = \sum_{i=1}^{n} PF(Q_i, \hat{T}_i, \tilde{p}_i, \tilde{h}_i, \tilde{\pi}_i)$$

Therefore the complete mathematical model of the multi-product inventory system with random replenishment under space and shortage level constraints is

    Maximize $PF(\boldsymbol{Q}, \hat{\boldsymbol{T}}, \tilde{\boldsymbol{p}}, \tilde{\boldsymbol{h}}, \tilde{\boldsymbol{\pi}})$

    Subject to

$$\sum_{i=1}^{n} w_i Q_i \leq W$$

$$\Pr(\hat{T}_i > T_{0i}) = \int_{\frac{Q_i}{D_i}}^{T_{Max_i}} f_{T_i}(t_i)dt_i \leq 1 - S_i$$

$$Q_i \geq 0, \ i = 1, 2, \ldots, n.$$

(Since the shortages occur only when the cycle time is greater than T_{0i}, and that the lower limit of the service level is S_i.)

4. Model with an exponential distribution for $\hat{T}_i$ and TrFN for $\tilde{p}_i$, $\tilde{h}_i$ and $\tilde{\pi}_i$

In this subsection we discuss the random replenishment model in the fuzzy sense.

If we assume that the time between replenishments is exponentially distributed with λ_i arrival rate, fuzzy expected profit in this model is given by

$$\text{Maximize } PF(\boldsymbol{Q}, \hat{\boldsymbol{T}}, \tilde{\boldsymbol{p}}, \tilde{\boldsymbol{h}}, \tilde{\boldsymbol{\pi}}) = \sum_{i=1}^{n} \left\{ (s_i - \tilde{p}_i)R_i - \tilde{h}_i I_i - (s_i - \tilde{p}_i)L_i - \tilde{\pi}_i B_i \right\}$$

$$= \sum_{i=1}^{n} \left\{ \frac{1}{\lambda_i} [D_i(1-\beta_i)(\tilde{p}_i - s_i) - \tilde{\pi}_i \beta_i D_i] e^{-\left(\frac{Q_i}{D_i}\right)\lambda_i} + \frac{1}{\lambda_i}[D_i(\tilde{p}_i - s_i) - \tilde{h}_i Q_i] + \frac{\tilde{h}_i Q_i}{\lambda_i^2}[1 - e^{-\left(\frac{Q_i}{D_i}\right)\lambda_i}] \right\} \quad (1)$$

Subject to

$$\sum_{i=1}^{n} w_i Q_i \leq W$$

$$e^{-\left(\frac{Q_i}{D_i}\right)\lambda_i} \leq 1 - S_i$$

$$Q_i \geq 0, \ i = 1, 2, \ldots, n.$$

Now, let us consider the inventory costs, $\tilde{p}_i$, $\tilde{h}_i$ and $\tilde{\pi}_i$ are imprecise in nature and expressed by trapezoidal fuzzy numbers (TrFNs). A TrFN, for example, $\tilde{p}_i = (p_{i1}, p_{i2}, p_{i3}, p_{i4})$, satisfying the condition $0 \leq p_{i1} \leq p_{i2} \leq p_{i3} \leq p_{i4}$ and has the following membership function:

$$\mu_{\tilde{p}_i}(x) = \begin{cases} \dfrac{x - p_{i1}}{p_{i2} - p_{i1}} & \text{if } p_{i1} \leq x < p_{i2} \\ 1 & \text{if } p_{i2} \leq x \leq p_{i3} \\ \dfrac{p_{i4} - x}{p_{i4} - p_{i3}} & \text{if } p_{i3} < x \leq p_{i4} \\ 0 & \text{otherwise} \end{cases}$$

Hence by using the fuzzy arithmetic operations by function principle, the fuzzy expected profit reduces to a trapezoidal fuzzy number

$$\widetilde{PF} = (PF_1, PF_2, PF_3, PF_4).$$

Here PF_1, PF_2, PF_3, PF_4 are all positive real valued function of Q_i ($i = 1, 2, \ldots, n$) satisfying the conditions $PF_1 \leq PF_2 \leq PF_3 \leq PF_4$. Using the functional principle the expressions of PF_r ($r = 1, 2, 3, 4$) are as follows:

$$PF_r = \sum_{i=1}^{n} \left\{ \frac{1}{\lambda_i} [D_i(1-\beta_i)(p_{ri}-s_i) - \pi_{(4-r+1)i}\beta_i D_i] e^{-\left(\frac{Q_i}{D_i}\right)\lambda_i} + \frac{1}{\lambda_i}[D_i(p_{ri}-s_i) - h_{(4-r+1)i}Q_i] + \frac{h_{ri}Q_i}{\lambda_i^2}[1 - e^{-\left(\frac{Q_i}{D_i}\right)\lambda_i}] \right\}.$$

Case 1: In this case we maximize the optimistic value of $\widetilde{PF}$ with predefined value α_1. Then the problem reduces to

$$\underset{Q}{Max} \ Max \ X_1$$

Subject to $m_{\rho_1}\left(\widetilde{PF} \geq X_1\right) \geq \alpha_1$

Using lemma 2.1 the above fuzzy constraint optimization problem can be transformed to the equivalent crisp problem as

$$\underset{Q}{Max} \begin{cases} \dfrac{\alpha_1(PF_3 - PF_4)}{\rho_1} + PF_4 & if \ \alpha_1 \leq \rho_1 \\ \dfrac{(1-\alpha_1)(PF_2 - PF_1)}{(1-\rho_1)} + PF_1 & if \ \alpha_1 > \rho_1 \end{cases}$$

Subject to

$$\sum_{i=1}^{n} w_i Q_i \leq W$$

$$e^{-\left(\frac{Q_i}{D_i}\right)\lambda_i} \leq 1 - S_i$$

$$Q_i \geq 0, \ i = 1, 2, \ldots, n.$$

Case 2: In this case we maximize the pessimistic value of $\widetilde{PF}$ with predefined value α_2. Then the problem reduces to

$$\underset{Q}{Max} \ Min \ X_2$$

Subject to $m_{\rho_2}\left(\widetilde{PF} \leq X_2\right) \geq \alpha_2$

Using lemma 2.1 the above fuzzy constraint optimization problem can be transformed to the equivalent crisp problem as

$$\underset{Q}{Min} \begin{cases} \dfrac{\alpha_2(PF_2 - PF_1)}{\rho_2} + PF_1 & if \ \alpha_2 \leq \rho_2 \\ \dfrac{(1-\alpha_2)(PF_3 - PF_4)}{(1-\rho_2)} + PF_4 & if \ \alpha_2 > \rho_2 \end{cases}$$

Subject to

$$\sum_{i=1}^{n} w_i Q_i \leq W$$

$$e^{-\left(\frac{Q_i}{D_i}\right)\lambda_i} \leq 1 - S_i$$

$$Q_i \geq 0, \ i = 1, 2, \ldots, n.$$

5. Numerical Illustration

For the illustration purpose we present a multi-product inventory problem with three products and the data are given in table 5.1. Here we consider the imprecise purchase cost inventory holding cost and shortage cost as trapezoidal fuzzy numbers.

Table 5.1. Input data

Items	D_i	β_i	λ_i	s_i	p_i	h_i	π_i	w_i	S_i
1	30	0.5	1/25	125	(82, 85, 90, 98)	(2, 2.2, 2.5, 2.7)	(5, 6, 8, 9)	3	0.55
2	25	0.6	1/40	140	(94, 97, 100, 102)	(2.5, 2.8, 3, 3.2)	(6, 8, 9, 10)	4	0.5
3	20	0.55	1/30	120	(90, 93, 95, 98)	(1.7, 2, 2.2, 2.5)	(4, 5, 7, 8)	3	0.6
W = 6000									

Using theses values, the problem (I) has been solved using a non-linear optimization technique (GRG method) for different values of α_j and ρ_j (j = 1, 2) and the results are presented in Table 5.2 to Table 5.5.

Table 5.2. Variations in ρ_1 and α_1 ($\alpha_1 \leq \rho_1$)

ρ_1 / α_1		0.25	0.5	0.75	1
0.2		64306.2	76372.7	80394.8	82405.9
0.5	X_1		**58273.1**	68328.4	73356.1
0.7				60284.1	67322.9
0.95					**59781.3**

Table 5.3. Variations in ρ_1 and α_1 ($\alpha_1 > \rho_1$)

ρ_1 / α_1		0	0.2	0.5	0.75
0.2		25964.0			
0.5	X_1	17194.1	20848.2		
0.7		11347.6	13539.9	20117.3	
0.95		**4039.1**	4404.5	5500.7	8424.1

92

Table 5.4. Variations in ρ_2 and α_2 ($\alpha_2 \leq \rho_2$)

ρ_2 $\downarrow \alpha_2$		0.25	0.5	0.75	1
0.2		25964.0	14270.7	10372.9	8424.1
0.5			**31810.7**	22066.2	17194.0
0.7	X_2			29861.8	23040.7
0.95					**30349.1**

Table 5.5. Variations in ρ_2 and α_2 ($\alpha_2 > \rho_2$)

ρ_2 $\downarrow \alpha_2$		0.25	0.5	0.75	1
0.2		64306.2			
0.5		73356.1	69585.3		
0.7	X_2	79389.3	77126.8	70339.5	
0.95		**86930.8**	86553.8	85422.5	82405.9

6. Discussion

Table 5.2 and Table 5.3 show the maximum optimistic value of the objective function measured in method 1. It is observed from the tables that if α_1 increases for each fixed value of ρ_1, the value of the objective function decreases while increasing the value of ρ_1 for each fixed value of α_1, the value of the objective function increases.

Since for the parameter $\rho_1 = 1$ and $\rho_1 = 0$, m_{ρ_1} - measure degenerates to the possibility and necessity measures respectively, we have the optimal values of the objective function at different confidence level α_1. For example, at $\alpha_1 = 0.95$ the optimistic and pessimistic values of the objective function are 59781.3 and 4039.1.

Similarly, Table 5.4 and Table 5.5 show the minimum optimistic value of the objective function measured in method 2. It is observed from the tables that if α_2 increase for each fixed value of ρ_2, the value of the objective function increases while increasing the value of ρ_2 for each fixed value of α_2, the value of the objective function decreases.

At $\alpha_2 = 0.95$ the optimistic and pessimistic values of the objective function are 86930.8 and 30349.1.

7. Conclusion and Future Scope

Now-a-days, to tackle the real world uncertain inventory systems the fuzzy set theory and the probability theory has been used very nicely. In this paper, a fuzzy stochastic inventory problem proposed for the random replenishment intervals and imprecise inventory costs using fuzzy set theory and probability theory complementary. After de-randomization fuzzy arithmetic operation under function principle, the optimistic and pessimistic values of the objective function are obtained. A possibility and necessity measure produces a balance between optimism and pessimism. Finally, the critical values of fuzzy objective function with respect to m_ρ - measure are obtained and the

result indicates that the confidence interval is wide in optimistic case as compare to pessimistic case.

There are several possible directions for future research. This work can be directly extended to consider some other distributions like uniform, normal etc. for replenishment intervals. It is also interesting to consider the problems with variable demand or the items received are not all perfect.

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Appendix

Let $\widetilde{A} = (a_1, a_2, a_3, a_4)$ and $\widetilde{B} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers then the fuzzy arithmetical operations under function principle are as follows:

1. Addition: $\widetilde{A} + \widetilde{B} = \widetilde{C}$, where the membership function of $\widetilde{C}$ is

$$\mu_{\widetilde{C}}(z) = \begin{cases} \dfrac{z - (a_1 + b_1)}{(a_2 + b_2) - (a_1 + b_1)} & \text{if } (a_1 + b_1) \leq z < (a_2 + b_2) \\ 1 & \text{if } (a_2 + b_2) \leq z < (a_3 + b_3) \\ \dfrac{(a_4 + b_4) - z}{(a_4 + b_4) - (a_3 + b_3)} & \text{if } (a_3 + b_3) \leq z < (a_4 + b_4) \\ 0 & \text{otherwise} \end{cases}$$

where a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4 are any real numbers.

2. Multiplication: $\tilde{A} \cdot \tilde{B} = \tilde{C}$, where the membership function of $\tilde{C}$ is

$$\mu_{\tilde{C}}(z) = \begin{cases} \dfrac{z - a_1 b_1}{a_2 b_2 - a_1 b_1} & \text{if } a_1 b_1 \leq z < a_2 b_2 \\ 1 & \text{if } a_2 b_2 \leq z < a_3 b_3 \\ \dfrac{a_4 b_4 - z}{a_4 b_4 - a_3 b_3} & \text{if } a_3 b_3 \leq z < a_4 b_4 \\ 0 & \text{otherwise} \end{cases}$$

where a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4 are all non zero positive real numbers.

3. Subtraction: $\tilde{A} - \tilde{B} = \tilde{C}$, where the membership function of $\tilde{C}$ is

$$\mu_{\tilde{C}}(z) = \begin{cases} \dfrac{z - (a_1 - b_4)}{(a_2 - b_3) - (a_1 - b_4)} & \text{if } (a_1 - b_4) \leq z < (a_2 - b_3) \\ 1 & \text{if } (a_2 - b_3) \leq z < (a_3 - b_2) \\ \dfrac{(a_4 - b_1) - z}{(a_4 - b_1) - (a_3 - b_2)} & \text{if } (a_3 - b_2) \leq z < (a_4 - b_1) \\ 0 & \text{otherwise} \end{cases}$$

where a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4 are any real numbers.

4. Division: $\tilde{A} / \tilde{B} = \tilde{C}$, where the membership function of $\tilde{C}$ is

$$\mu_{\tilde{C}}(z) = \begin{cases} \dfrac{z - (a_1 / b_4)}{(a_2 / b_3) - (a_1 / b_4)} & \text{if } (a_1 / b_4) \leq z < (a_2 / b_3) \\ 1 & \text{if } (a_2 / b_3) \leq z < (a_3 / b_2) \\ \dfrac{(a_4 / b_1) - z}{(a_4 / b_1) - (a_3 / b_2)} & \text{if } (a_3 / b_2) \leq z < (a_4 / b_1) \\ 0 & \text{otherwise} \end{cases}$$

where a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4 are all non zero positive real numbers.