M.TECH/CSE/2ND SEM/CSEN 5201/2022

ADVANCED ALGORITHMS
(CSEN 5201)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) Which of the following algorithms solve single source shortest path problem?
(a) Dijkstra's Algorithm (b) Ford-Fulkerson Algorithm
(c) Prim's Algorithm (d) DFS algorithm

(ii) A new number can be inserted into a min-heap of n (**>** 0) numbers stored in an array in time
(a) O(n) (b) O(n lg n) (c) O(lg n) (d) $O(n^2)$.

(iii) The augmenting path in a flow network
(a) determines the edge connectivity of a network
(b) comprises edges which can admit positive flow
(c) converts a tree network into a cyclic chain
(d) is used only for cyclic chain networks.

(iv) Which of the following is not true about comparison based sorting algorithms for sorting n numbers?
(a) The minimum possible time complexity of a comparison based sorting algorithm is O(n log n) for a random input array
(b) Some comparison based sorting algorithms have worst-case time complexity $O(n^2)$
(c) Counting Sort is not a comparison based sorting algorithm
(d) Heap Sort is not a comparison based sorting algorithm.

(v)
```
for i = 1 to m
{
   for j = 1 to i
      sum = sum + A [ i ][ j ];
}
```
The time complexity of the above code snippet in the worst case is
(a) $O(m^2)$ (b) $O(m^3)$ (c) O(m) (d) None of these.

(vi) An algorithm is made up of 2 modules M1 & M2. If M1 is O(f(n)), and that of M2 is O(g(n), then the order of the algorithm is
(a) O(max(f(n), g(n))) (b) O(min(f(n), g(n)))
(c) O(f(n) + g(n)) (d) O(f(n) * g(n)).

(vii) Which of the following is the worst-case time complexity of BFS algorithm when the graph G = (V, E) is implemented by adjacency list
(a) O(|V|+|E|) (b) $O(|V|^2)$ (c) O(|V|*|E|) (d) none of these.

(viii) Height of a heap containing n elements is
(a) $\lfloor \lg n \rfloor$ (b) $\lceil \lg n \rceil$ (c) n (d) None of these.

(ix) Let P(n) denote the worst-case running time and Q(n) the average-case running time of an algorithm on an input of size n. Which one of the following alternatives is *always true*?
(a) P(n) = Ω(Q(n)) (b) P(n) = θ(Q(n))
(c) P(n) = O(Q(n)) (d) P(n) = o(Q(n))

(x) The recurrence relation T(1) = 2, T(n) = 3T(n/4) + n has the solution equal to
(a) O(n) (b) O(logn) (c) $O(n^{¾})$ (d) None of the above.

Group – B

2. (a) Write a recursive algorithm to solve Towers of Hanoi problem and then analyse the time complexity of your algorithm. [(CO4)(Apply/IOCQ)]

(b) Let's assume that a polynomial of degree m is represented as $t(n) = \sum_{i=0}^{m} a_i n^i$ **.** Prove that $t(n) = O(n^m)$. [(CO3)(Apply/IOCQ)]

(4 + 4) + 4 = 12

3. (a) An array A contains n **>** 0 distinct positive integers. We want to find the k^{th} smallest integer in A for different values of k. Describe an algorithm that is known to do this in O(n) expected time for any k, 1 ≤ k ≤ n.
[(CO3)(Understand/HOCQ)]

(b) Formulate Merge Sort algorithm as a Divide-and-Conquer approach. Analyze the performance of this algorithm in worst case. [(CO3)(Apply/IOCQ)]

(c) Heapsort and Mergesort have the same worst-case time complexities. Is there any reason why Heapsort may be considered better compared to Mergesort?
[(CO5)(Evaluate/HOCQ)]

3 + (3 + 3) + 3 = 12

Group - C

4. (a) G = (V,E) is a undirected graph with 7 vertices numbered 1 through 7 and the following 12 edges in the form (vertex, vertex, **edge cost**) with the edge cost in bold: (1,2,**2**), (1,3,**4**), (1,4,**1**), (2,4,**3**), (2,5,**10**), (3,4,**2**), (3,6,**6**), (4,5,**7**), (4,6,**8**),

(4,7,**4**), (5,7,**6**), (6,7,**1**). Draw the graph and represent the graph as adjacency matrix. [(CO2)(Remember/LOCQ)]

(b) Using Prim's algorithm, find out a spanning tree of the graph. Demonstrate every step to show how the spanning tree grows based on your proposed algorithm. [(CO3)(Apply/LOCQ)]

(c) What would be the time complexity of Prim's algorithm? [(CO1)(Remember/HOCQ)]

(1 + 2) + 7 + 2 = 12

5. (a) Suppose G = (V, E) is a weighted, directed graph. Give a priority based implementation of Dijkstra's algorithm to solve single-source shortest path problem. What is the time complexity of this algorithm in the worst case? [(CO2)(Understand/LOCQ)]

(b) Suppose a graph contains a cycle with negative weight edge. What problem would you face when you are trying to find a shortest path between a source & a destination? Justify your answer. [(CO5)(Evaluate/HOCQ)]

(7 + 1) + 4 = 12

Group - D

6. (a) How Dynamic Programming approach is different from Divide and Conquer strategy? [(CO2)(Remember/LOCQ)]

(b) A 0-1 Knapsack Problem has a knapsack of weight limit 12, and 4 items with the values and weights given below:

Item	1	2	3	4
Value	8	9	7	5
Weight	6	7	5	3

Solve the problem using dynamic programming and obtain the maximum total value that can be achieved. How will the solution change if it is specified that item 4 must be included in the knapsack? [(CO3)(Apply/HOCQ)]

(c) Solve the Fractional Knapsack Problem with the data given in (a). Show the change in the solution if it is specified that item 4 must be included entirely (i.e., fully, not partially) in the knapsack. [(CO3)(Apply/LOCQ)]

2 + 6 + 4 = 12

7. (a) Give an algorithm for finding shortest path between every pair of vertices in a given graph G= (V, E) with positive arc costs. [(CO2)(Remember/LOCQ)]

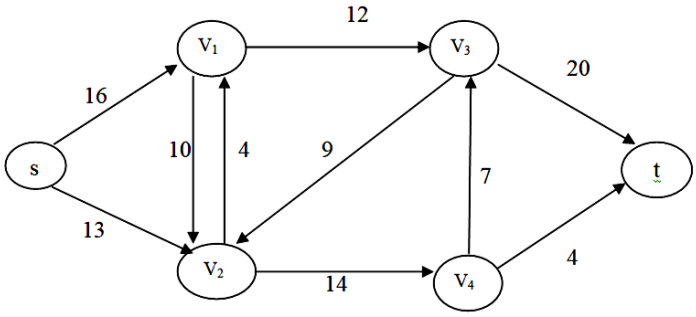
(b) Find an optimal parenthesization of a matrix-chain product using dynamic programming method where the dimensions of the matrices are (30,35), (35,15), (15,5), (5,10), (10,20), (20,25). [(CO3)(Apply/IOCQ)]

7 + 5 = 12

Group – E

8. (a) What do you mean by maximum-flow problem? Define an augmenting path in a residual network. [(CO1)(Remember/LOCQ)]

(b) Consider the following flow network:



Apply FORD-FULKERSON algorithm on the above flow network to find the maximum flow in the network. s & t denote the source & destination respectively and the capacity associated with every edge is given along with the edge. What is the running time of this algorithm in the worst case considering arbitrary (positive) capacity values? [(CO3)(Apply/IOCQ)]

(2 + 2) + (6 + 2) = 12

9. (a) Differentiate clearly between the sets **P**, **NP** and **NPC**. Give an example of a problem in **NP** that is not known to be in **P** at the current time. [(CO2)(Understand/IOCQ)]

(b) Assuming that the CLIQUE problem is in **NPC**, how can we show that the VERTEX COVER problem is also in **NPC**? Explain the logic of the proof briefly. [(CO1)(Remember/HOCQ)]

(c) Give a polynomial-time 2-approximation algorithm for vertex cover problem. [IOCQ]

4 + 5 + 3 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	36.46	39.58	23.96

Course Outcome (CO):

After the completion of the course, students will be able to –

CO1. Remember time complexities of various existing algorithms in different situations
CO2. Understand the basic principles of different paradigms of designing algorithms
CO3. Apply mathematical principles to solve various problems
CO4. Analyze the complexities of various algorithms
CO5. Evaluate the performance of various algorithms in best case, worst case and average case
CO6. Create/Design a good algorithm for a new problem given to him/her.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question