DATA STRUCTURES AND ALGORITHMS
(MCAP 1201)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) Which of the following is not $O(n^2)$?
(a) $n^{1.97}$ (b) 12^{10} n + 1999 (c) $n^3 / \sqrt{n}$ (d) 2^{20} n.

(ii) Consider the polynomial $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3$, where $a_i \neq 0$, for all i. The minimum number of multiplications needed to evaluate p on an input x is
(a) 3 (b) 4 (c) 6 (d) 9.

(iii) An algorithm consists of two modules X1, X2. Their order is f(n) and g(n), respectively. The order of the algorithm is
(a) Max(f(n),g(n)) (b) Min(f(n),g(n))
(c) f(n)+g(n) (d) f(n)*g(n).

(iv) Given an empty stack, after performing
push (1), push (2), pop, push (3), push(4), pop, pop, push (5), pop, what is the value at the top of the stack?
(a) 4 (b) 3 (c) 2 (d) 1.

(v) In a full binary tree if there are L leaves, then total number of nodes N is
(a) N = 2*L (b) N = L + 1 (c) N = L – 1 (d) N = 2*L – 1.

(vi) A binary tree with 27 nodes has _____ null branches.
(a) 54 (b) 27 (c) 26 (d) 28.

(vii) Given a pointer to the first node of a singly linked list, which of the following operation can be implemented in O(1) time?
I) Insertion at the front of the linked list
II) Insertion at the end of the linked list
III) Deletion of the front node of the linked list
IV) Deletion of the last node of the linked list
(a) I and II (b) I and III (c) I, II and III (d) I, II and IV.

(viii) Which of the following is not a stable sorting algorithm?
(a) Insertion sort (b) Selection sort
(c) Bubble sort (d) Merge sort.

(ix) A hash function f defined as f (key) = key mod 7, with linear probing used to resolve collisions. Insert the keys 37, 38, 72, 48, 98 and 11 into the table indexed from 0 to 6.What will be the location of 11?
(a) 3 (b) 4 (c) 5 (d) 6.

(x) The postftx equivalent of the prefix * + *ab -cd* is
(a) *ab + cd -** (b) *abcd + -** (c) *ab + cd *-* (d) *ab + -cd **.

Group- B

2. (a) Consider the following declaration for a two-dimensional array in JAVA char a[100][100]; Assuming that the main memory is byte addressable and that the array is stored starting from memory address 0, find the address of a[40][50]. [(CO1)(Apply/IOCQ)]

(b) An n x n array A is defined as follows: A [i][j] = i – j for all i, j, $0 \le i \le n - 1$, $0 \le j \le n - 1$
Find the sum of the elements of the array A. [(CO1)(Apply/IOCQ)]

(c) Write an algorithm with O(n) complexity to find the sum of all possible pairs in an array of N elements.
For example if the input is arr[]={1,2}. Then all possible pairs are (1, 1), (1, 2), (2, 1) and (2, 2). So, the required sum in that case is (1 + 1 + 1 + 2 + 2 + 1 + 2 + 2) i.e. 12. [(CO6)(Create/HOCQ)]

3 + 3 + 6 = 12

3. (a) Represent the following equations in Big-Oh notation.
(i) $T(n) = 2T(n-1)+c$, $T(1)=1$
(ii) $T(n) = T(n/2)+c$, $T(1)=1$; [(CO2)(Apply/IOCQ)]

(b) Define O, Ω and Θ symbols in relation to growth rate of a function. State some limitations of Big O notation. [(CO2)(Remember/LOCQ)]

(c) Represent the polynomial $5x^5 + 4x^2 – 25x + 10$ with array(s). [(CO5)(Apply/IOCQ)]

6 + (3 + 1) + 2 = 12

Group - C

4. (a) Are recursive routines more efficient than non-recursive ones? Justify your answer with examples. [(CO3)(Analyse/IOCQ)]

(b) Convert the expression given below into its corresponding postfix expression using a stack.
((5 + 3) * 4) – 2 / 3 ^ 2 [(CO4)(Apply/IOCQ)]

(c) State the Towers of Hanoi problem. Devise a solution to the Towers of Hanoi problem clearly explaining the precondition and post condition for the algorithm. [(CO4)(Apply/IOCQ)]

3 + 3 + 6 = 12

5. (a) Write a recursive function to find the nth Fibonacci number and derive its time complexity. [(CO3)(Apply/IOCQ)]

(b) A single array A [MAXSIZE] is used to implement two stacks. The two stacks grow from opposite ends of the array. Variables top1 and top2 (top1 < top2) point to the location of the topmost element in each of the stacks. If the space is to be used efficiently, write the condition to check whether the stack is full. Justify your answer. [(CO5)(Evaluate/HOCQ)]

(c) What are the advantages of circular queue over linear queue? Write insert and serve functions for a circular queue. [(CO4)(Understand/LOCQ)]

4 + 2 + (3 + 3) = 12

Group - D

6. (a) Consider the In-order and Post-order traversals of a tree as given below:
In-order: j e n k o p b f a c l g m d h i
Post-order: j n o p k e f b c l m g h i d a
Draw the tree. What will be the pre-order traversal sequence of the tree? [(CO1)(Apply/IOCQ)]

(b) Write a function to calculate the number of *external nodes* in a binary search tree. [(CO1)(Apply/IOCQ)]

(c) Determine the number of internal nodes in a full binary tree of height *k*. [(CO1)(Evaluate/HOCQ)]

5 + 3 + 4 = 12

7. (a) Construct B-tree of the set {2,6,1,8,3,4,5,10,7,15,14,16,13}. Minimum order of the tree is 2. Clearly mention each step of the B-tree construction. [(CO1)(Apply/IOCQ)]

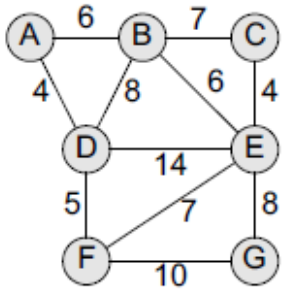
(b) Construct AVL tree of the following set of elements {2,6,1,8,3,4,5,10,7,15,14}
Clearly mention the different rotations used and balance factor of each node. [(CO1)(Apply/IOCQ)]

(c) What are the advantages of AVL tree over binary search tree? [(CO5)(Analyze/IOCQ)]

5 + 5 + 2 = 12

Group - E

8. (a) Construct a minimum spanning tree of the following graph using Prims algorithm. Let the Starting vertex be D. [(CO3)(Apply/IOCQ)]



(b) Write an algorithm for binary search using recursion and discuss its time complexity. [(CO2)(Understand/LOCQ)]

(c) Sort the following set using selection sort showing resulting set after each iteration:

26, 33, 35, 29, 19, 12, 22 [(CO4)(Apply/IOCQ)]

5 + 4 + 3 = 12

9. (a) Suppose that a hash table contains hash_size = 13 entries indexed from 0 through 12 and that the following keys are to be mapped into the table:

10, 100, 32, 45, 58, 126, 3, 29, 200, 400, 0

Determine the hash addresses and find how many collisions occur when these keys are reduced by applying the operation % hash_size. [(CO5)(Apply/IOCQ)]

(b) Describe the general idea of linear search. How is the idea of sentinel used to increase the efficiency of linear search? What are the best, worst and average case complexities of linear search? [(CO2)(Analyse/IOCQ)

5 + (2 + 2 + 3) = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	14.58	72.92	12.5

Course Outcome (CO):

After the completion of the course students will be able to

1. Define the basic data structures and their operations.
2. Evaluate algorithms in terms of time and memory complexity of basic operations.
3. Understand the significance and context of application of different algorithm design techniques.
4. Understand the significance and utility of different data structures and the context of their application.
5. Evaluate solutions of a problem with different data structures and thereby understand how to select suitable data structures for a solution.
6. Create or design an algorithm to aid in the solution of a real world problem.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question