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HERITAGE INSTITUTE OF TECHNOLOGY

M.Tech 1st Semester Examination, 2014 Session : 2014-2015

Discipline : AEIE

Paper Code : MATH5101 Paper Name : Advanced Mathematical Methods

Time Allotted : 3 hrs Full Marks : 70

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and
any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: 10 x 1=10

(i) If $T:U \to V$ is any linear transformation from U to V, then
 (a) the ker T is a subspace of U (b) the ker T is a subspace of V
 (c) the range of T is a subspace of U (d) none of these.

(ii) The eigen values of the matrix $\begin{bmatrix} 3 & 17 & 26 \\ 0 & 15 & 39 \\ 0 & 0 & -2 \end{bmatrix}$ are
 (a) 3, 0, 2 (b) 54, 31, 23
 (c) 3, 15, -2 (d) -6, 9, 13

(iii) If V is a finite dimensional vector space and S is a basis of V, then
 (a) number of elements of S = dimension of V,
 (b) number of elements of S > dimension of V,
 (c) number of elements of S < dimension of V,
 (d) none of these.

(iv) For a general non-linear programming problem Kuhn-Tucker conditions are
 (a) sufficient (b) necessary
 (c) necessary and sufficient (d) not applicable

(v) Which of the following is not a linear transformation from $\Re^3$ to $\Re^3$?
 (a) $T(x,y,z)=(x,2y,3x-y)$, (b) $T(x,y,z)=(2x,2y,5z)$,
 (c) $T(x,y,z)=(-x+y+z, x-y+z, x+y-z, x+y+z)$,
 (d) $T(x,y,z)=(x,y,0)$.

(vi) Eigen values of a Skew –Hermitian Matrix will be
 (a) 0 (b) equal
 (c) either purely imaginary or 0 (d) purely imaginary

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(vii) The *duality gap* in LPP is
(a) 0 (b) infinite
(c) less than zero (d) 1

(viii) If V and W are finite dimensional vector spaces over a field F and $T:V \to W$ is a linear mapping, then
(a) Nullity of T + rank of T = dim V, (b) Nullity of T + rank of T > dim V,
(c) Nullity of T + rank of T < dim V, (d) none of these.

(ix) Consider the following two bases of E = {(1,0,0), (0,1,0), (0,0,1)} and S = {(1,0,1), (2,1,2), (1,2,2)}. Then the change-of-basis matrix P from the basis E to the basis S is

(a) $\begin{bmatrix} 0 & 1 & 1 \\ 2 & 1 & 3 \\ 4 & 2 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & -3 \\ 4 & 0 & 2 \end{bmatrix}$

(x) The function $f(x,y) = x^2y + y^2 + 2y$ has
(a) a maximum point, (b) a minimum point,
(c) a saddle point, (d) none of these.

Group – B

2.(a) Examine if the set S is a subspace of $\Re^3$, where $S = \{(x,y,z) \in \Re^3 : x = 0\}$.

(b) Prove that, the set $S = \{(2,1,1),(1,2,1),(1,1,2)\}$ is a basis of $\Re^3$. 6+6=12

3.(a) Determine whether or not the set of vectors S= $\{(1,1,1),(1,2,3),(2,-1,1)\}$ forms a basis of R^3.

(b) Apply the Gram-Schmidt orthogonalization process to find an orthogonal basis for the subspace U of R^4 spanned by S= $\{(1,1,1,1),(1,2,4,5),(1,-3,-4,-2)\}$ 6+6=12

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Group – C

4.(a) A mapping $T:\Re^3 \to \Re^3$ is defined by

$T(x_1,x_2,x_3)=(x_1+x_2+x_3, 2x_1+x_2+2x_3, x_1+2x_2+x_3), (x_1,x_2,x_3)\in\Re^3$.

Show that T is a linear mapping.

(b) Find Ker T and its dimension.

6+6=12

5.(a) Let V be the vector space of n-square real matrices. Let M be an arbitrary but fixed matrix in V. Let f: V$\to$V be defined by f(A)=AM+MA, where A is any matrix in V. Show that f is linear.

(b) Show that the functions f(t) = sint, g(t)= cost and h(t) = t from R into R is linearly independent. 6+6=12

Group – D

6. Solve the following non-linear programming problem using Lagrange multiplier method:

Maximize $f(x_1,x_2) = 2x_1+x_2+10$

Subject to $x_1+2x_2^2=3$

Also find the effect of changing the right-hand side of the constraint on the optimum value of f. 8+4=12

7. Minimize $f = x_1^2 + x_2^2 + x_3^2 + 40x_1 + 20x_2$

Subject to

$x_1 \geq 50,$

$x_1 + x_2 \geq 100,$

$x_1 + x_2 + x_3 \geq 150$

by applying Kuhn-Tucker conditions. 12

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Group - E

8.(a) Consider the following problem:

Maximize $z = x_1 + x_2 + 3x_3$

Subject to
$$3x_1 + 2x_2 + x_3 \le 3,$$
$$2x_1 + x_2 + 2x_3 \le 2,$$
$$x_1, x_2, x_3 \ge 0.$$

Solve the problem by Simplex method.

(b) Construct the dual of the following LPP:

Maximize $z = 3x_1 + 4x_2,$

Subject to
$$x_1 + x_2 \le 12,$$
$$2x_1 + 3x_2 \le 21,$$
$$x_1 \le 8,$$
$$x_2 \le 6,$$
$$x_1, x_2 \ge 0.$$

10+2=12

9. Solve the following LPP using Big-M method:

Maximize $Z = x_1+2x_2+3x_3-x_4$

Subject to $x_1+2x_2+3x_3=15$

$2x_1+x_2+5x_3 =20$

$x_1+2x_2+x_3+x_4=10$

$x_1,x_2,x_3,x_4 \ge 0$

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